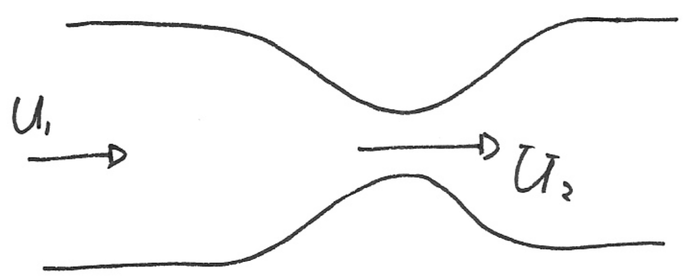
Bernoulli:

1




$$P_1 + \tfrac{1}{2}\rho_a U_1^2 = P_2 + \tfrac{1}{2}\rho_a U_2^2$$

$$U_1 A_1 = U_2 A_2$$

So

$$P_1 - P_2 = \tfrac{1}{2}\rho_a\left[U_2^2 - U_1^2\right] = \tfrac{1}{2}\rho_a U_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

Manometer height

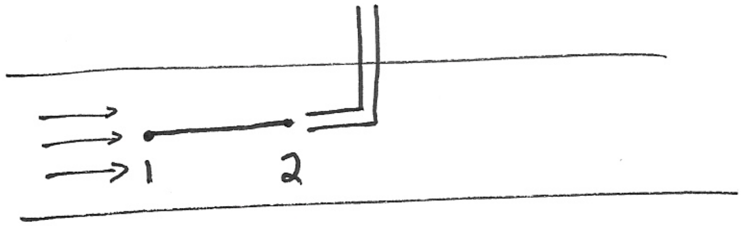
$$\rho_m g H = \Delta P = \tfrac{1}{2}\rho_{air} U_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

So

$$\boxed{U_1 = \sqrt{\frac{2\rho_m g H}{\rho_{air}\left(\frac{A_1^2}{A_2^2} - 1\right)}}}$$

# Bernoulli

(2)




$U_2 = 0$

So from B $\Rightarrow$ $P_1 + \frac{1}{2}\rho_a U^2 = P_2$

$$P_2 - P_1 = \frac{1}{2}\rho_a U^2$$

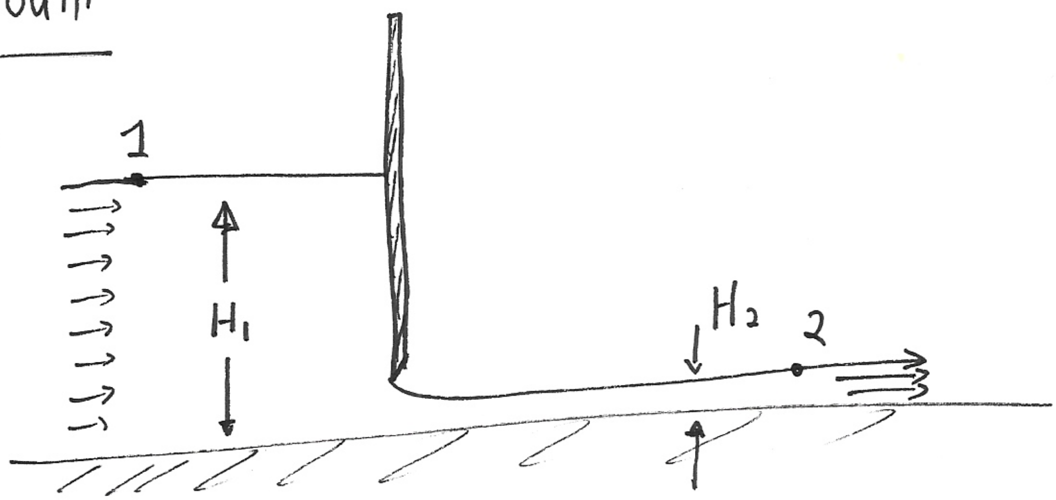
Manometer

$$\rho_m g H = \Delta P = \frac{1}{2}\rho_{air} U^2$$

$$U = \sqrt{\frac{2\rho_m g H}{\rho_{air}}}$$

# Bernoulli

(3)




Mass

$$U_1 H_1 = U_2 H_2$$

B/

$$P_1 + \tfrac{1}{2}\rho U_1^2 + \rho g H_1 = P_2 + \tfrac{1}{2}\rho U_2^2 + \rho g H_2$$

$$P_1 = P_2 = P_{ATM}$$

So/

$$(H_1 - H_2)\rho g = \tfrac{1}{2}\cancel{\rho}(U_2^2 - U_1^2) = U_1^2\left(\frac{H_1^2}{H_2^2} - 1\right)$$

$$2g(H_1 - H_2) = U_1^2\left(\frac{H_1^2 - H_1^2}{H_2^2}\right)$$

$$\frac{2g}{U_1^2} = \frac{(H_1 + H_2)\cancel{(H_1 - H_2)}}{H_2^2\,\cancel{(H_1 - H_2)}} = \frac{H_1 + H_2}{H_2^2}$$

$$\boxed{U_1^2 = \frac{2 g H_2^2}{H_1 + H_2}}$$

Bernoulli:

4

From #1 $$P_2 - P_\infty = \frac{1}{2}\rho_{air} U_1^2 \left( \frac{A_1^2}{A_2^2} - 1 \right)$$

Liquid Flow

$$P_\infty - P_2 = Q_L R = Q_L \frac{128 \mu L}{\pi D^4}$$

$$Q_L \left( \frac{128 \mu L}{\pi D^4} \right) = \frac{1}{2}\rho_{air} U_1^2 A_1^2 \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right)$$

$$Q_L = \frac{\frac{1}{2}\rho_{air} Q_{Air}^2 \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right)}{128 \mu L / \pi D^4}$$

Mass

$$\frac{\partial}{\partial r}(r v) = 0$$

Momentum

$$\rho v \frac{\partial v}{\partial r} + \frac{\partial P}{\partial r} = \mu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial}{\partial r}\left(\tfrac{1}{2}\rho v^2 + P\right) = \mu \frac{\partial^2 v}{\partial z^2}$$

From mass

$$Q = 2\pi r h v$$

So $\boxed{v = \dfrac{Q}{2\pi r h}}$ Velocity is known!

If we neglect viscosity

$$P + \tfrac{1}{2}\rho v^2 = \text{Const.}$$

when $r = R_0$ $\quad P = P_\infty$ $\quad v_{R_0} = \dfrac{Q}{2\pi R_0 h}$

$$P + \tfrac{1}{2}\rho v^2 = P_\infty + \tfrac{1}{2}\rho v_{R_0}^2$$

$$P - P_\infty = \tfrac{1}{2}\rho \left( v_{(r)}^2 - v_{R_0}^2 \right)$$

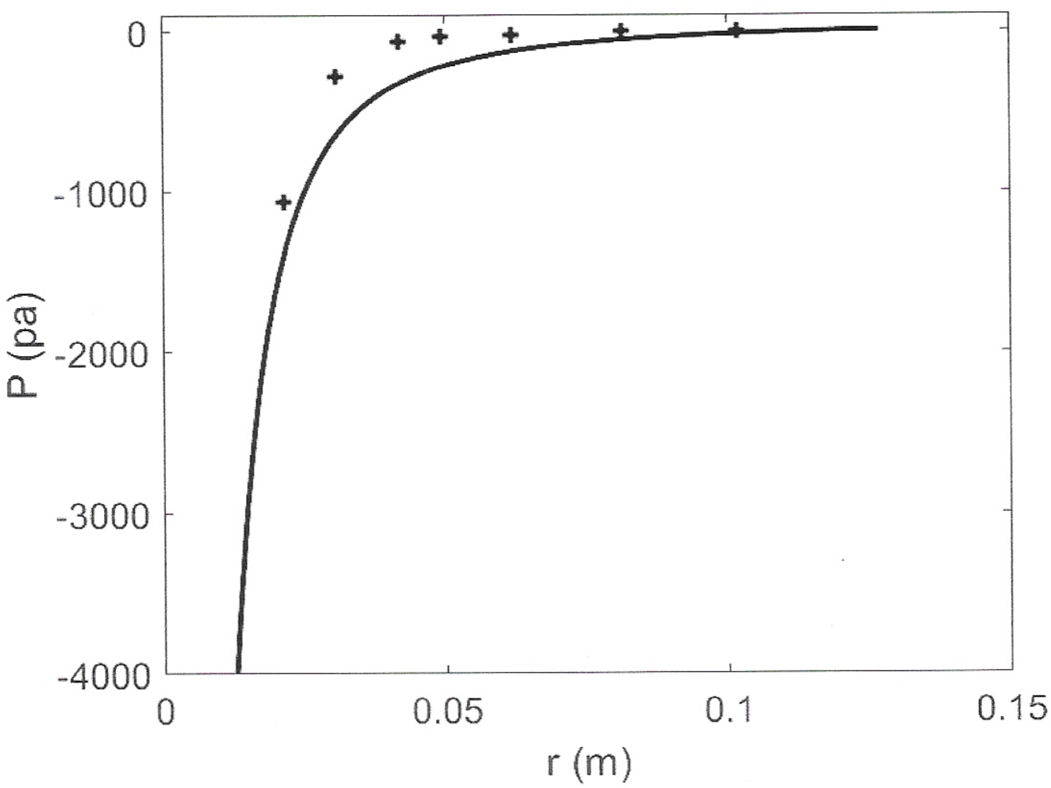
Compare to data (next page) looks good!

total force

$$F = 2\pi \int_{R_{in}}^{R_{out}} P(r)\, dr$$

Put in #'s and we get 0.37 kg,

Pretty much exact



0
-1000
-2000
-3000
-4000
P (pa)
0
0.05
0.1
0.15
r (m)

With Viscosity

$$\frac{\partial}{\partial r}\left(P + \tfrac{1}{2}\rho v^2\right) = -\frac{\rho v^2 f}{4f} = -\frac{\rho f}{4h}\left(\frac{Q}{2\pi h}\right)^2 \frac{1}{r^2}$$

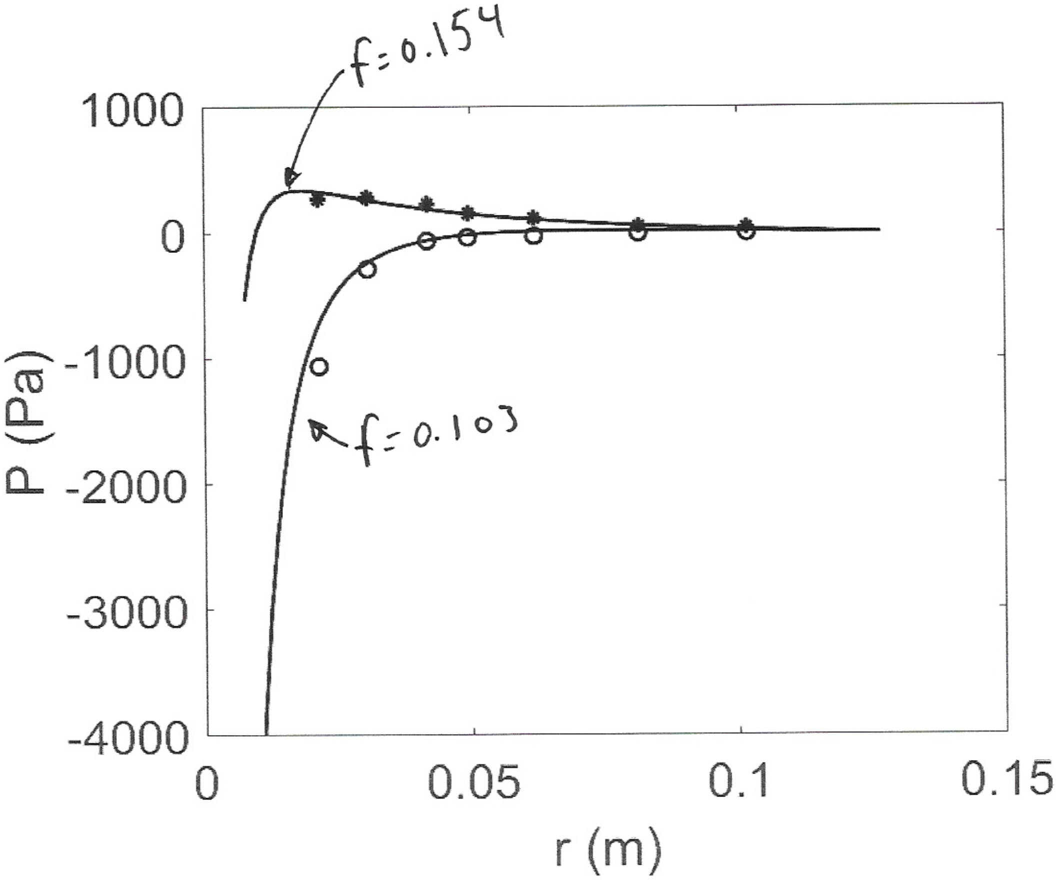
Integrate

$$P + \tfrac{1}{2}\rho v^2 = \frac{\rho f}{4h}\left(\frac{Q}{2\pi h}\right)^2 \frac{1}{r} + C$$

$$P + \tfrac{1}{2}\rho v^2 = \frac{\rho f}{4h} v^2 r + C$$

$$\boxed{P + \tfrac{1}{2}\rho v^2\left(1 - \frac{f}{2h} r\right) = \text{Constant}}$$

When we can fit f, data looks great!


f=0.154
f=0.103
1000
0
-1000
-2000
-3000
-4000
P (Pa)
0
0.05
0.1
0.15
r (m)

## Converging nozzle simulation

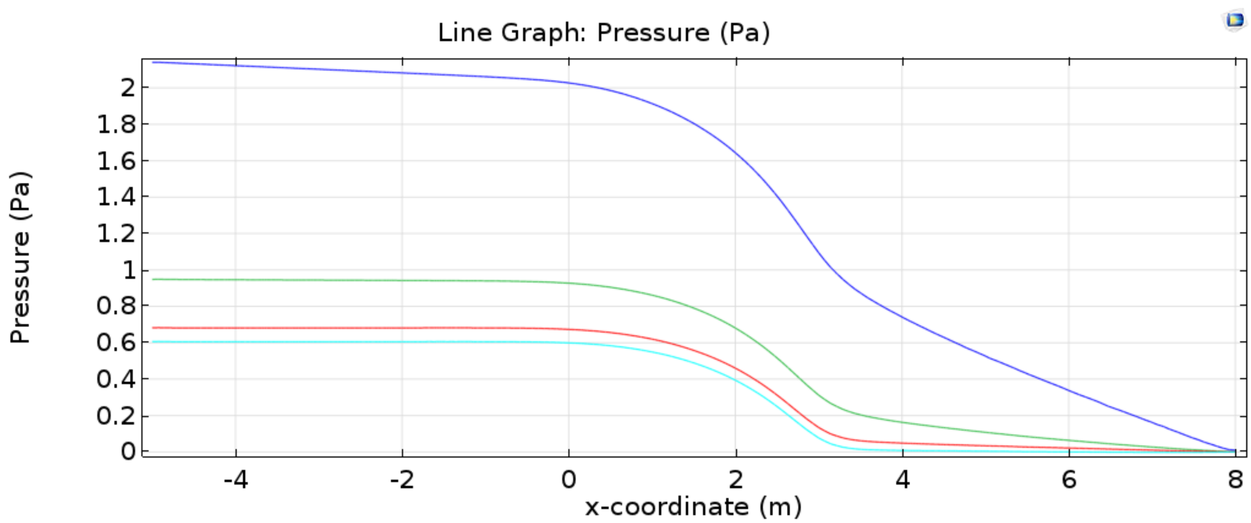



*Figure 1: Pressure versus distance down the nozzle at Re=100, 1000,10000, and 10000 from top to bottom. Here pressure is plotted along the center streamline. The behavior at Re=100,000 is starting to converge to that would be predicted by Bernoulli's equation.*

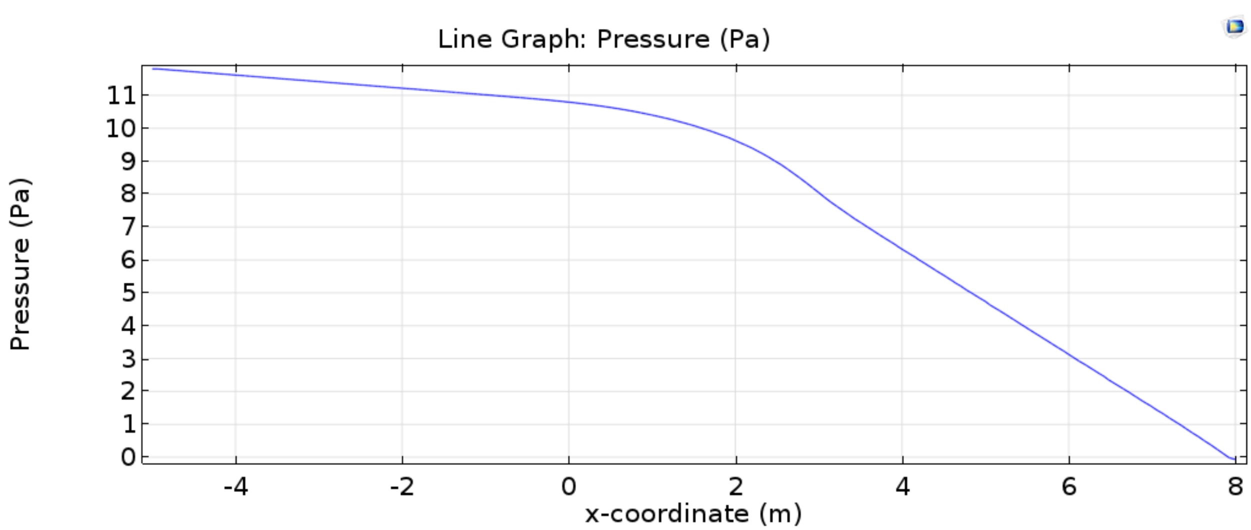



*Figure 2: Same as above, only at Re=10. Here the effect of viscosity clearly shows a very different behavior than at high Reynolds number. The behavior we see here looks like hydraulic resistors in series.*

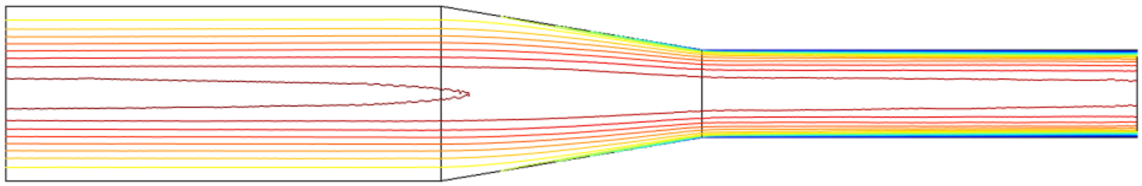


*Figure 3: For Re=10,000 we plot contours of the Bernoulli constant* $P+\frac{1}{2}\rho v^2$*, . We see the contours look nearly like the streamlines.*

## Lift on a flat plate.

| Angle | Cd (Re=1000) | CL (Re=1000) | CD (Re=10,000) | CL (Re=10,000) | CL (Theoretical) |
|---|---|---|---|---|---|
| 0 | 0.0857 | 0.00039 | 0.0436 | 0.001 | 0 |
| 1 | 0.0859 | 0.0937 | 0.0444 | 0.118 | 0.10960 |
| 2 | 0.0874 | 0.194 | 0.0465 | 0.234 | 0.21917 |
| 3 | 0.0895 | 0.287 | 0.0506 | 0.349 | 0.32867 |
| 4 | 0.0933 | 0.382 | 0.0577 | 0.463 | 0.43807 |
| 5 | 0.0989 | 0.479 | 0.0698 | 0.576 | 0.54734 |
| 6 | 0.108 | 0.515 | 0.086 | 0.675 | 0.65643 |
| 7 | 0.118 | 0.465 | 0.109 | 0.740 | 0.76534 |
| 8 | 0.125 | 0.415 | 0.131 | 0.749 | 0.87401 |
| 9 | 0.135 | 0.398 | 0.146 | 0.728 | 0.98241 |
| 10 | 0.146 | 0.407 | 0.16 | 0.701 | 1.09051 |

*Figure 4: Values of lift and drag from Comsol simulation. For Re=1000 we used the laminar, stationary solver. For Re=10,000 we used the k-e turbulence model with the default settings. For the higher angle of attack we needed to use the transient solver to get a converged solution. Generating all the data took some time for the higher Re situation.*

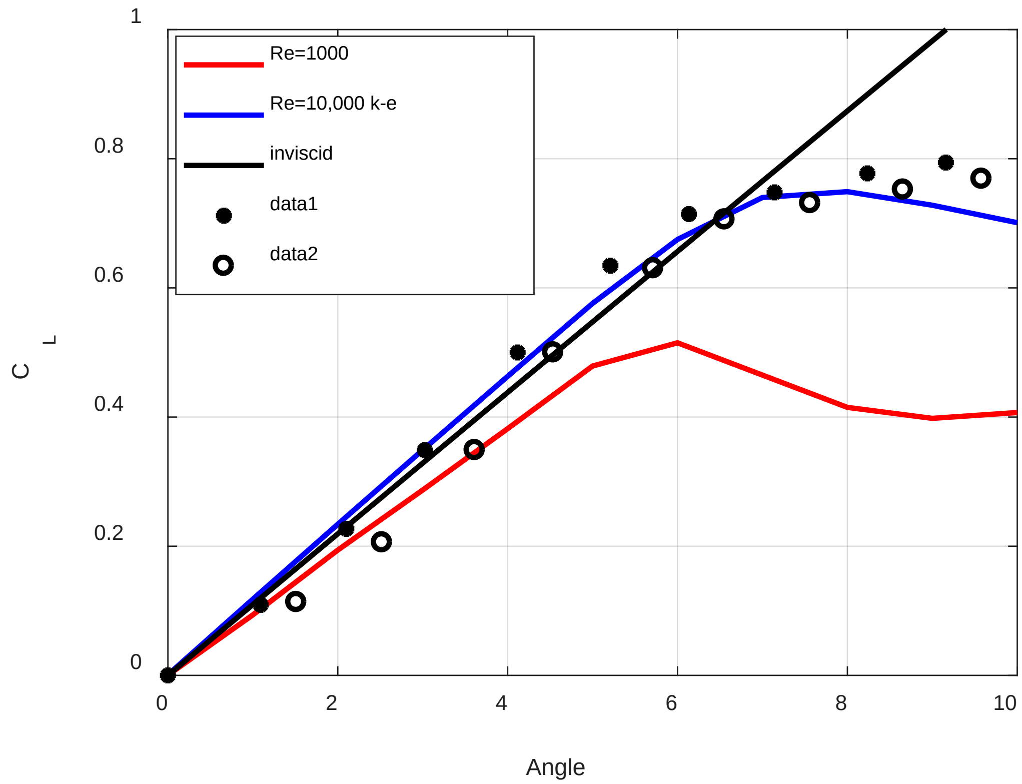



*Figure 5: Plot of the data from the table shown previously for lift on a flat plate. The black like is the theoretical result from inviscid theory. Points are data taken at Re=40,000.*

# BL #1

Plane, 30,000ft   $U \approx 250 \frac{m}{s}$

Assume

Wing $\approx$ 2 m from leading edge.

$\rho_{30,000ft} \approx 0.5 \ kg/m^3$

$\mu_{30,000ft} \approx 1.5 \times 10^{-5} \ Pa \cdot s$

Transition occurs  $Re \approx 10^5$

$$10^5 = \frac{\rho U L}{\mu} = \frac{(0.5)(250)L}{1.5 \times 10^{-5}}$$

$L \approx 12$ mm , distance from leading edge where flow goes turbulent.

For BL thickness, turbulent

$$\frac{\delta}{x} = \frac{0.37}{Re^{1/5}}$$

wing ($L = 2m$) $\Rightarrow Re = 1.6 \times 10^7$

fuselage ($L = 40m$) $\Rightarrow Re = 3.3 \times 10^8$

BL thickness.

$$\boxed{\begin{aligned} \delta_{wing} &\approx 2.6 \text{ cm} \\ \delta_{fuse} &\approx 30 \text{ cm}. \end{aligned}}$$

BL #2

Assume

$U = 15\ m/s$

$\rho = 1.2\ kg/m^3$

$\mu = 1.8 \times 10^{-5}\ Pa \cdot s$

$L \approx 1.5\ m$

$$Re = \frac{\rho U L}{\mu} = 1.5 \times 10^6$$

becomes turbulent when $Re \approx 10^5$

$$10^5 = \frac{(1.2)(15)\ L}{1.8 \times 10^{-5}} \Rightarrow \underline{L = 10\ cm.}$$

$$\frac{\delta_{Hood}}{x} = \frac{0.37}{Re^{1/5}} \Rightarrow \boxed{\delta_{Hood} = 3.2\ cm}$$

# BL #3

Water $\rho = 1000\ kg/m^3$

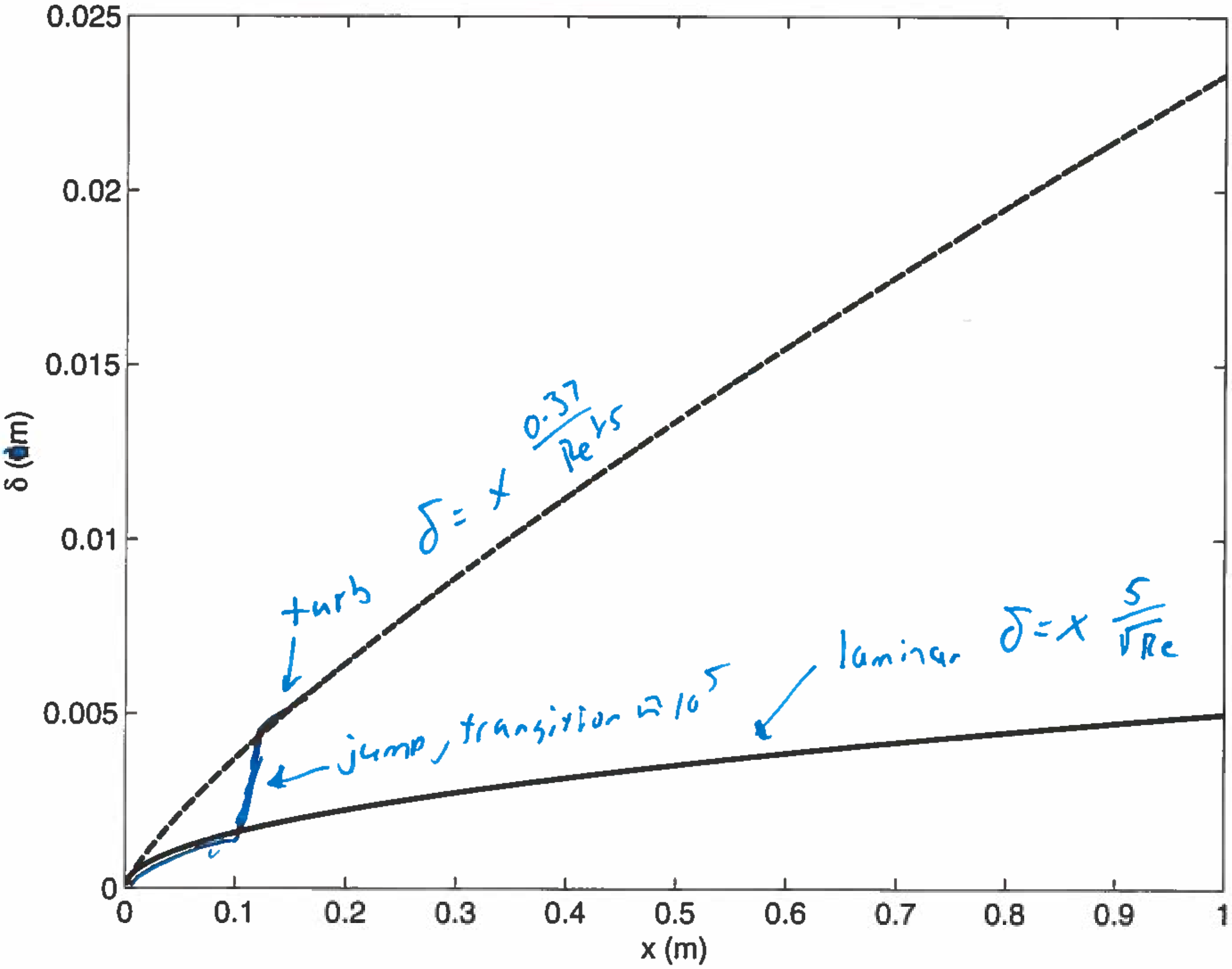
$\mu = 10^{-3}\ Pa\text{-}s$

$$Re = 10^5 = \frac{\rho U x}{\mu} = \frac{(1000)(1\ \tfrac{m}{s})\ x}{10^{-3}}$$

transition

$$\boxed{x = 0.1\ m}$$

See plot.



0.025
0.02
0.015
0.01
0.005
0
δ (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x (m)
turb $\delta = x \frac{0.37}{Re^{1/5}}$
laminar $\delta = x \frac{5}{\sqrt{Re}}$
jump, transition $\approx 10^5$

#4

To start, let's assume turbulent over whole plate, since laminar is only first 10%

$$C_f = \frac{\tau(x)}{\frac{1}{2}\rho U^2} = \frac{0.0592}{Re_x^{1/5}} = \frac{0.0592}{\left(\frac{\rho U x}{\mu}\right)^{1/5}}$$

$$C_d = \frac{1}{L}\int_0^L C_f\,dx = \frac{5}{4} C_f = \frac{5}{4}\cdot\frac{0.0592}{Re^{1/5}}$$

When $Re_L = 10^6$

$$C_d = 0.0047$$

$$\frac{\text{Force}}{\text{width}} = C_d \tfrac{1}{2}\rho U^2 = (0.0047)\tfrac{1}{2}(1000)\,1^2$$

$$\approx 2.35 \frac{N}{m}$$

$$\boxed{\text{Force} \approx 2.35\ N}$$

$$\text{Force} = \int_0^L \tau(x)$$

$$= \frac{1}{2}\rho U^2 \left( \int_0^{L_1} \frac{0.664}{\sqrt{\frac{\rho U x}{\mu}}}\, dx + \int_{L_1}^{L} \frac{0.0592}{Re_x^{1/5}}\, dx \right)$$

$$= \frac{1}{2}\rho U^2 \left( \frac{1.328\, L_1}{\sqrt{Re_{L_1}}} + \frac{0.074 \cdot L}{Re_L^{1/5}} \left( 1 - \left(\frac{L_1}{L}\right)^{4/5} \right) \right)$$

$L_1 = 0.1\ m$

$Re_{L_1} = 10^5$

$$= \frac{1}{2}\rho U^2 \left( 0.00042 + 0.0047 (1 - 0.16) \right)$$

$$= \frac{1}{2}\rho U^2 \underbrace{(0.0044)}_{C_D}$$

Note that if we account for mixed Laminar/turbulent flow, $C_D$ changes by ~10% which corresponds to first 10% being laminar in first simple calculation.