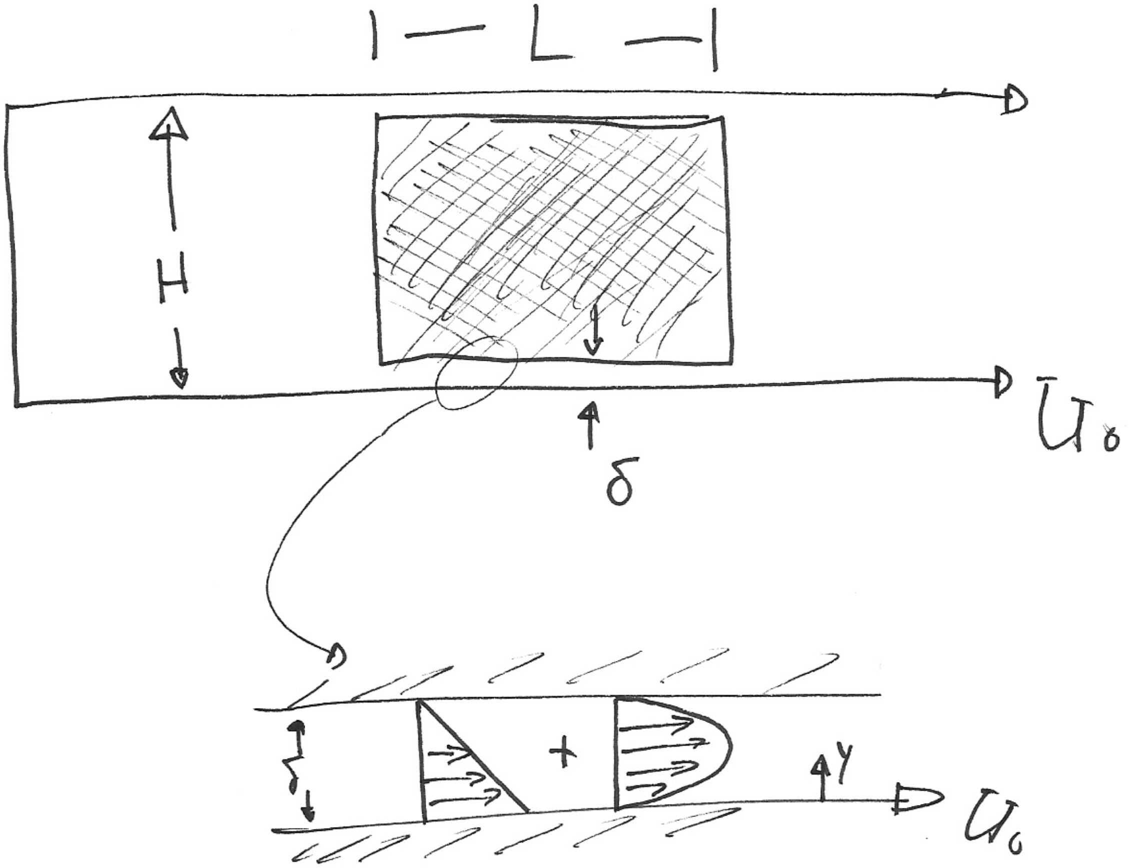
2D



$$U(y) = U_0\left(\frac{\delta - y}{\delta}\right) + \frac{\Delta P}{2\mu L}\, y(\delta - y) \quad \text{(From Book)}$$

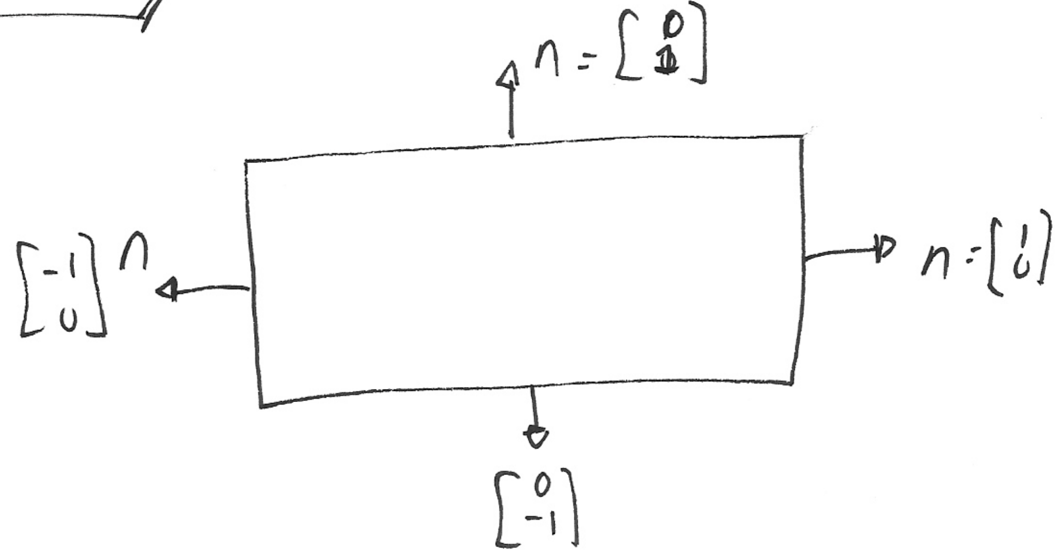
$$Q = \int_0^{\delta} U(y)\,dy$$

$$Q = 2\left(\frac{U_0\delta}{2} + \frac{\Delta P \delta^3}{12\mu L}\right) = U_0\delta + \frac{\Delta P \delta^3}{6\mu L}$$

$$Q = U_0 H \quad \text{(From Constraint of moving in a closed tube)}$$

$$U_0 H = U_0\delta + \frac{\Delta P \delta^3}{6\mu L} \Rightarrow \Delta P = \frac{6U\mu L(H-\delta)}{\delta^3}$$

2D- Force



$\underline{\underline{T}} \cdot \underline{n}$ on each face $\quad T = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix} + \mu \begin{bmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ & 2\frac{\partial v}{\partial y} \end{bmatrix}$

Left + Right

Since $u = 0$ and $v = 0$

Left

$$\underline{\underline{T}} \cdot \underline{n} = \begin{bmatrix} -P + 2\mu \frac{\partial u}{\partial x} & \mu \frac{\partial v}{\partial x} \\ \mu \frac{\partial v}{\partial x} & -P \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} P + 2\mu \frac{\partial u}{\partial x} \\ \mu \frac{\partial v}{\partial x} \end{bmatrix}$$

Right

$$\underline{n} \cdot \underline{\underline{T}} = -\begin{bmatrix} P + \mu \frac{\partial u}{\partial x} \\ \mu \frac{\partial v}{\partial x} \end{bmatrix}$$

if we plug in typical values $P \ggg \mu \frac{\partial u}{\partial x}$

$\boxed{\text{So for X force only worry about Pressure}}$

Upper Surface

$$u = v = 0 \quad \frac{\partial u}{\partial x} = 0 \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 0$$

$$T = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix} + \mu \begin{bmatrix} 0 & \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} & 0 \end{bmatrix}$$

$$\underline{\underline{T}} \cdot \underline{n} = \begin{bmatrix} -P & \mu \frac{\partial u}{\partial y} \\ \mu \frac{\partial u}{\partial y} & -P \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mu \frac{\partial u}{\partial y} \\ -P \end{bmatrix}$$

Lower Surface

$$\underline{\underline{T}} \cdot \underline{n} = \begin{bmatrix} -P & \mu \frac{\partial u}{\partial y} \\ \mu \frac{\partial u}{\partial y} & -P \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\mu \frac{\partial u}{\partial y} \\ P \end{bmatrix}$$

For x force $\mu \frac{\partial u}{\partial y}$ is the Stress

Note the Sign change. In lower gap $\frac{\partial u}{\partial y} < 0$ at the Block

In upper gap $\frac{\partial u}{\partial y} > 0$ at the block

Force is thus retarding the motion.

Lower gap

$$u(y) = U_0\left(\frac{\delta - y}{\delta}\right) + \frac{\Delta P}{2\mu L}\, y(\delta - y)$$

$$\left.\frac{\partial u}{\partial y}\right| = -\frac{U_0}{\delta} + \frac{\Delta P}{2\mu L}(\delta - 2y)$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = -\left(\frac{U_0}{\delta} + \frac{\Delta P\,\delta}{2\mu L}\right)$$

total Force

$$F = (H-\delta)\Delta P + 2\mu L\left(\frac{U_0}{\delta} + \frac{\Delta P\,\delta}{2\mu L}\right)$$

$$F = (H-2\delta)\Delta P + \frac{2\mu L U_0}{\delta} + \Delta P\,\delta$$

$$F = \Delta P\cdot(H-\delta) + \frac{2\mu L U_0}{\delta}$$

$$F = \frac{6U\mu L(H-\delta)^2}{\delta^3} + \frac{2\mu L U_0}{\delta}$$

Compare to Comsol

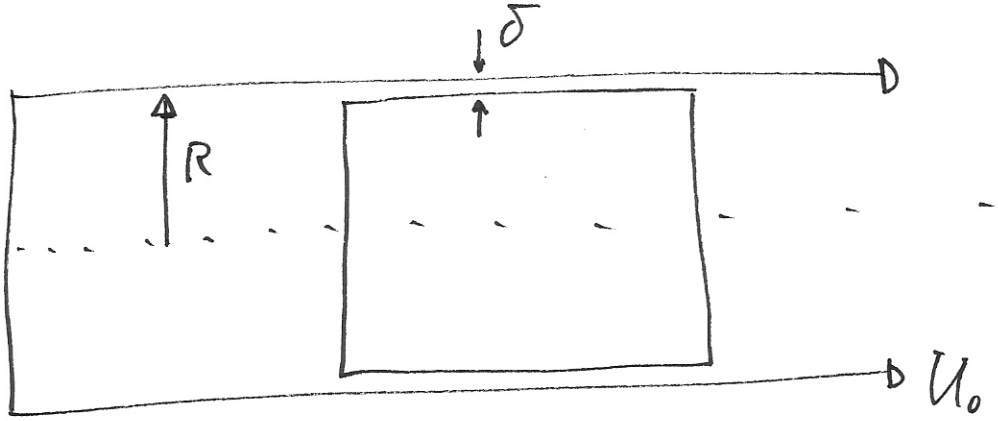
$L = 4$

$H = 1$

	ΔP_{Comsol}	$\Delta P = \frac{6\mu U L (H-\delta)}{\delta^3}$	F_{Comsol}	Analysis
$\delta = 0.1$	22173	21,600	19,875	19,520
$\delta = 0.05$	184,180	172,800	165,800	173,400

Seems pretty good!

Now Cylinder



Same basic procedure. Assume 2D flow is wrapped around the cylinder. $\bar{R} = R - \delta/2$

$$Q = \pi R^2 U_0 = Q_{2D}\, 2\pi \bar{R}$$

$$\cancel{\pi} R^2 U_0 = 2\cancel{\pi} \bar{R} \left(\frac{U_0 \delta}{2} + \frac{\Delta P \delta^3}{12 \mu L} \right)$$

$$\Delta P = \left(\frac{R^2}{\bar{R}} - \delta \right) \frac{6 \mu U_0 L}{\delta^3}$$

$$\Delta P = \left(\frac{R}{\cancel{R}(1 - \frac{\delta}{2}R)} - \delta \right) \frac{6 \mu U_0 L}{\delta^3}$$

$$\Delta P = \left(\frac{1}{1 - \frac{\delta}{2R}} - \frac{\delta}{R} \right) \frac{6 \mu U_0 L R}{\delta^3} \equiv \frac{6 \mu U L R}{\delta^3} \underbrace{A}_{\text{Define for later.}}$$

As in 2D, get the force

$$F = \Delta P \pi (R-\delta)^2 + 2\pi (R-\delta) L \mu \frac{\partial u}{\partial y}\Big|_{y=\delta}$$

$$F = \Delta P \pi (R-\delta)^2 + 2\pi (R-\delta) \mu L \left(\frac{U}{\delta} + \frac{\Delta P \delta}{2L} \right)$$

$$F = \Delta P \pi (R-\delta)^2 + \pi (R-\delta) \left[\frac{2\mu U L}{\delta} + \Delta P \delta \right]$$

Let's compare to Comsol to see how we are doing.

Set $U=1$, $L=4$, $\delta = 0.1$, $R = 1/2$

$\Delta P_{Comsol} = \underline{\underline{11,089}}$

$$\Delta P = \frac{6 U \mu L R}{\delta^3} \left(\frac{1}{1-\frac{\delta}{2R}} - \delta/R \right) = 12{,}000 (0.91) = \underline{\underline{10,932}}$$

$Force_{Comsol} = $ Pressure + Viscous $= \underline{5573} + \underline{1471}$

$$Force = \Delta P \pi (R-\delta)^2 + \pi (R-\delta) \left(\frac{2\mu U L}{\delta} + \Delta P \delta \right)$$

$= \underline{5495} + \underline{1473}$ ✓ Compares really well.

Now Overall force Balance.

$$F = \Delta P \pi (R-\delta)^2 + \pi (R-\delta) \left[\frac{2\mu U L}{\delta} + \Delta P \delta \right]$$

$$F = \Delta P \pi (R-\delta)^2 \left(1 + \frac{\delta}{R-\delta} \right) + \frac{2\mu U L}{\delta} \pi (R-\delta)$$

$$F = \frac{6\mu U L R}{\delta^3} A (R-\delta)^2 \pi \left[1 + \frac{\delta}{R-\delta} \right] + \frac{2\mu U L}{\delta} \pi (R-\delta)$$

$$F_{drag} = \frac{6\mu U L R \pi (R-\delta)^2}{\delta^3} \underbrace{\left[A\left(1 - \frac{\delta}{R-\delta}\right) + \frac{\delta^2}{3R(R-\delta)} \right]}_{B}$$

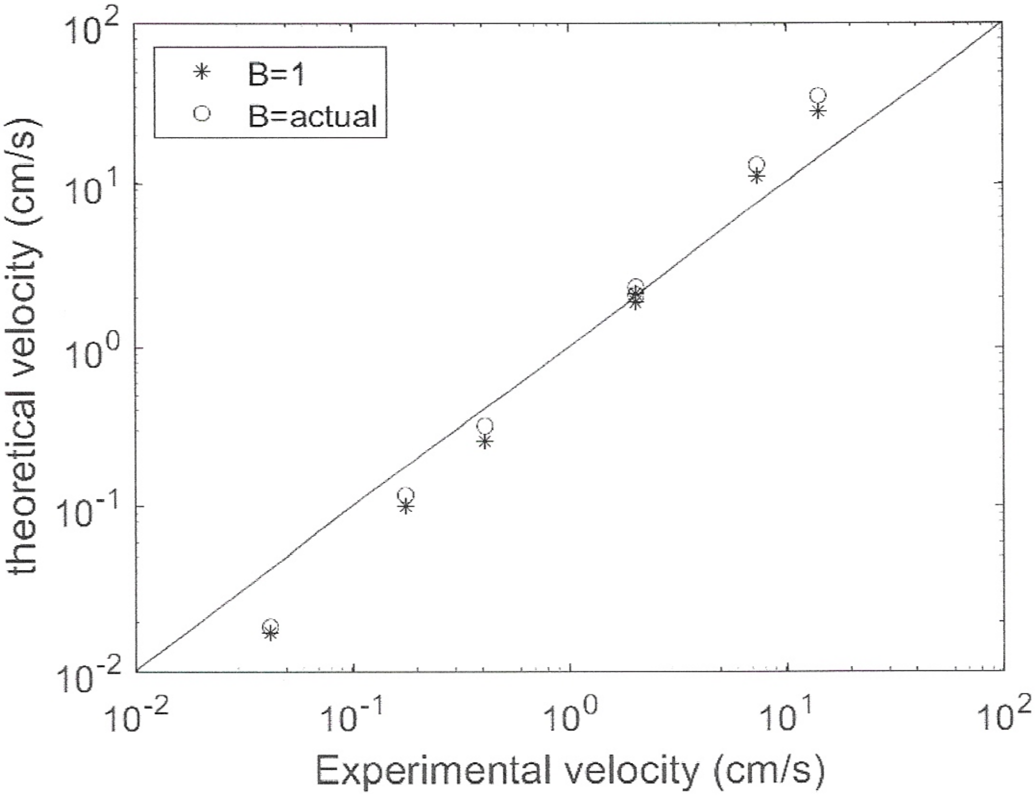
Note this # is close to 1.

$$F_{grav} = \pi (R-\delta)^2 (\rho_B - \rho_f) L g$$

$$(\rho_B - \rho_f) g = \frac{6\mu U R}{\delta^3} B$$

$$U = \frac{(\rho_b - \rho_f) g \delta^3}{6 \mu R B}$$

for Simplicity take Limit $B \to 1$

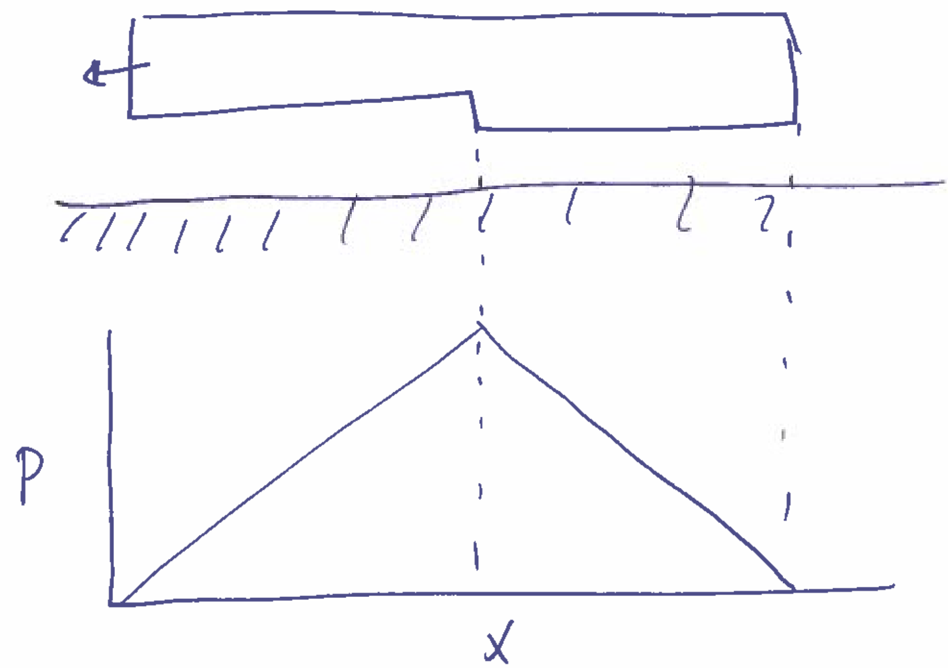


B=1
B=actual
theoretical velocity (cm/s)
Experimental velocity (cm/s)

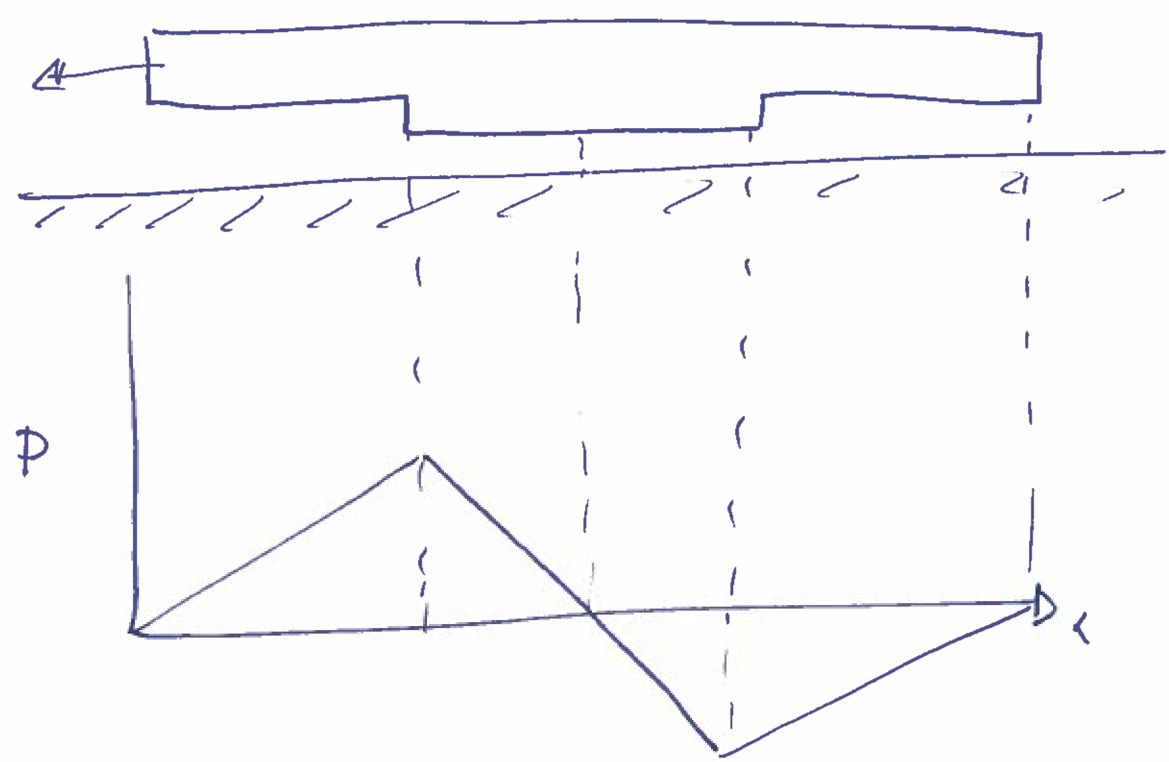
Slider

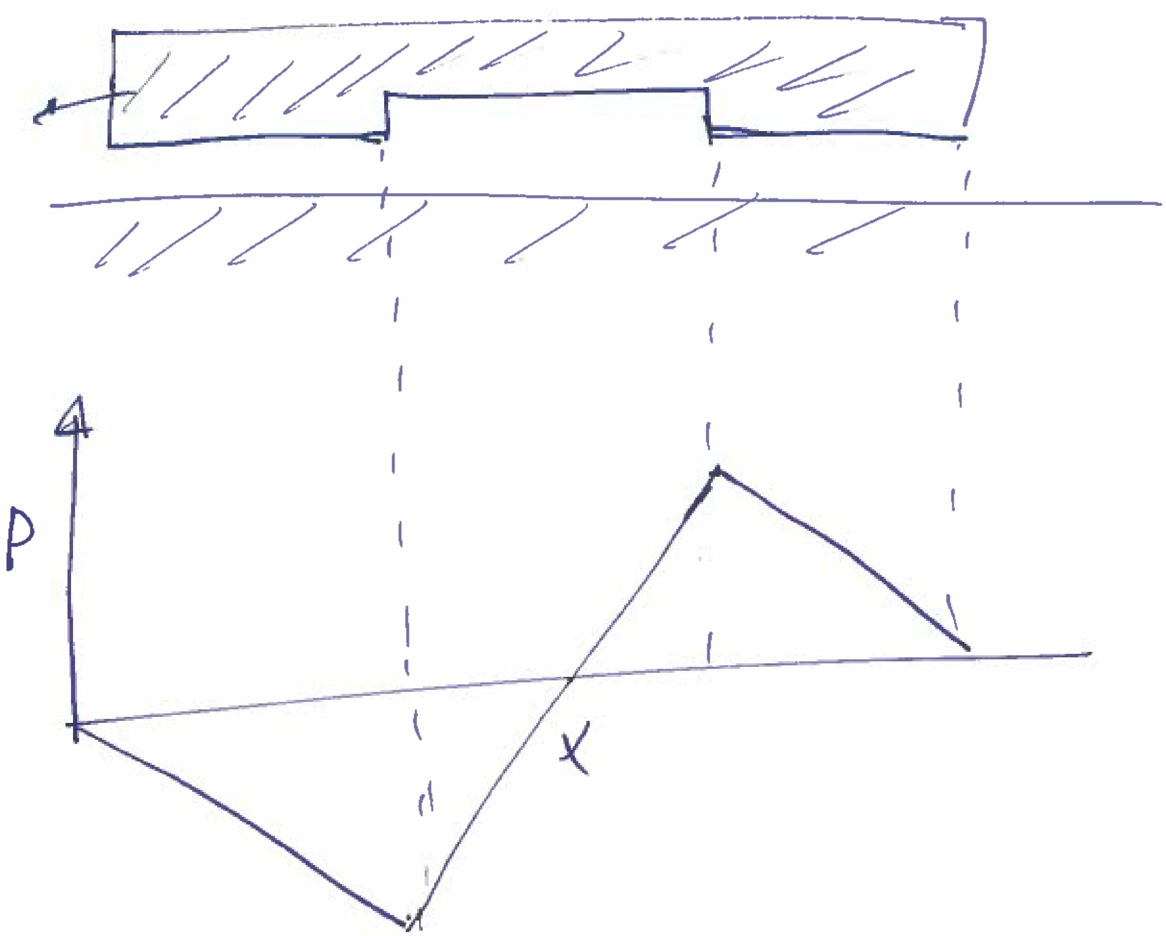
From Section 8.6

We found for the following case

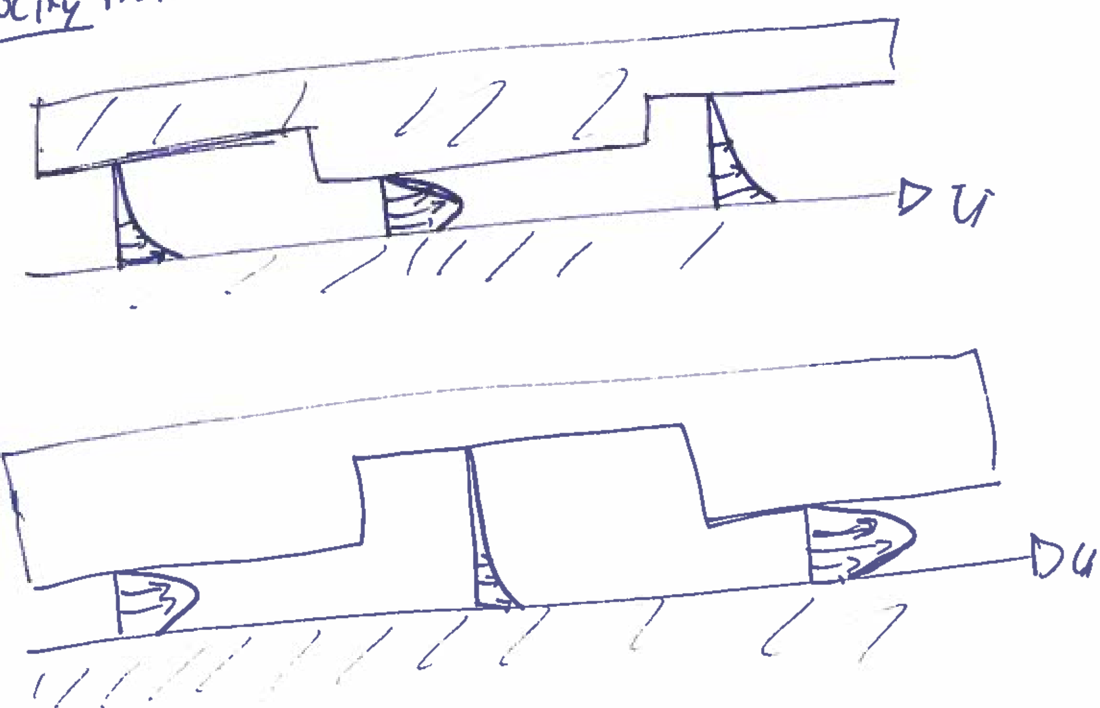


By Symmetry





Velocity field



Solution is equivalent to book where

$$L_1 = L$$
$$L_2 = L/2$$

Since solution goes through zero at midpoint

$$P\left(\frac{2}{L} + \frac{1}{L}\frac{H_1^3}{H_2^3}\right) = \frac{6\mu U}{H_2^3}(H_1 - H_2)$$

$$P = \frac{\frac{6\mu UL}{H_2^3}(H_1 - H_2)}{2 + H_1^3/H_2^3}$$

when $H_1/H_2 = 2$

$$P = \frac{\frac{6\mu UL}{H_1^{\cancel{3}2}}\frac{H_1^2}{H_2^2}\left(1 - H_2/H_1\right)}{2 + (H_1/H_2)^3} = \frac{6\mu UL}{H_1^2}\left(\frac{4(1-1/2)}{2+8}\right)$$

$$= \frac{6\mu UL}{H_1^2}\left(\frac{1}{5}\right)$$

when $H_1/H_2 = 1/2$

$$P = \frac{6\mu UL}{H_1^2}\left(\frac{\frac{1}{4}\left(1 - \frac{1}{2}\right)}{2 + \frac{1}{8}}\right)$$

$$P = \frac{-6\mu UL}{H_1^2}\left(\frac{2}{9}\right)$$

PROBLEM #1

Here is my calculation of the drag coefficient over a cylinder. The drag coefficient, I defined as

$$C_d = \frac{F}{\frac{1}{2}\rho U^2 D}$$

Where F is the computed force per unit width.

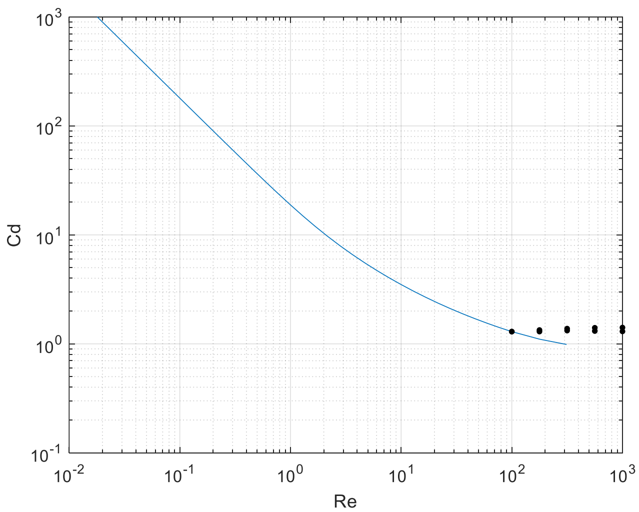


Figure 1: The solid line is the steady solver whereas the points are time dependent solver. The time dependent solver I plot the maximum and minimum drag force since it is oscillatory. Here the computational domain is 20 times the diameter of the cylinder.

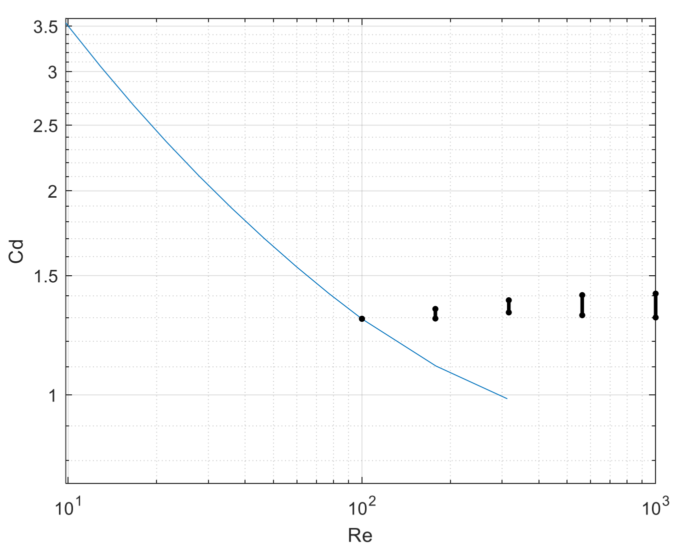


Figure 2: Zoomed in version of figure 1. For the time dependent solution I plot the range of unsteady drag forces. We see the unsteady behavior emerge from the steady solution at Re=100. The prediction is that the unsteady flow changes the trend we see in reducing Cd with Re.