Fluid flow and the Navier Stokes equations

I. VIDEO LECTURES

This week you should watch

- Videos for chapters 7 & 8 (Momentum, Stress Tensor, Newtonian Fluid)

II. READING

This week you should read

- Chapter 7 & 8

III. READING QUESTIONS

1. Explain, in words what the material derivative represents.
2. Why do we use a tensor to represent the state of stress of a material?
3. What is the physical meaning of the divergence of the stress tensor?
4. What is the differential form of conservation of mass for an incompressible flow?
5. When is the assumption of incompressible flow a good one?
6. What role does gravity play in a constant density flow?
7. What is the relationship between the stress tensor at a point and the stress vector acting on a surface passing through that point?
8. What's the difference between Euler's equations and the Navier-Stokes equations?
9. Physically, what is the reason that the stress tensor must be symmetric?
10. Physically, conceptually, and briefly, describe in words what is the constitutive law for a Newtonian fluid.
11. What is the boundary condition on the fluid velocity at a solid wall (state in words or mathematically)?
12. What is vorticity and what does it represent?
13. What are the units of the stress tensor?

IV. REYNOLDS NUMBER ESTIMATES

Estimate (order of magnitude) the Reynolds number for

- A car traveling at 55 mph.
- A hurricane (with 80 mph winds over 100 miles)
- A bacteria in water (2 μm in size moving at 100 μm/s)

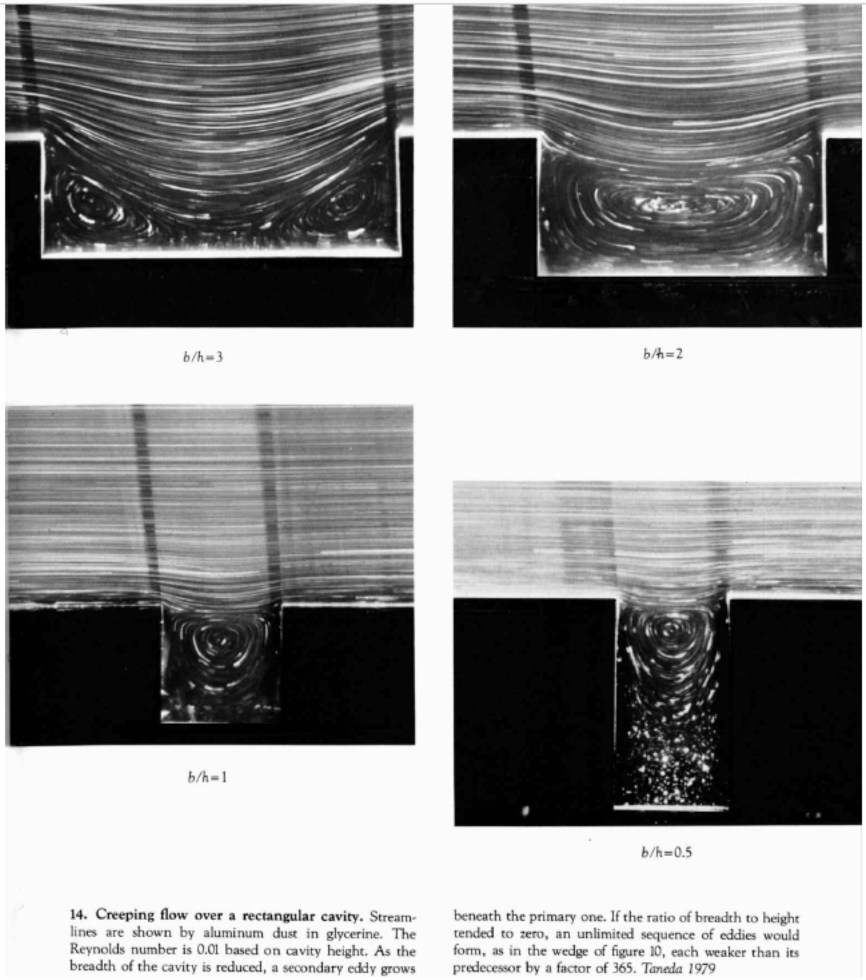

FIG. 1 Images from Album of Fluid motion.

V. COMSOL

1. See the images for creeping flow in a rectangular cavity from Album of Fluid Motion. Reproduce the experimental results in Figure 1. Include a panel of 4 images from Comsol to match the experiments (visually).

2. Consider laminar flow down a tube. At low Reynolds number, a neutrally buoyant particle placed at the the tube entrance will "go with the flow". Whatever streamline the particle starts on, the particle will stay on forever. Nothing interesting happens. It is a curious fact that a particle in a tube at "moderate" particle Reynolds number (i.e. greater than 1, but still laminar flow) will not stay on the streamline it starts on. As the particle traverses the tube, it will feel one force which will push it toward the wall and another that will repel it. The particle will then assume a fixed radial location in tube. If the particles come into the tube in random radial locations, the particles leave the tube confined to an annular ring; all particles are at the preferred radius. This result was first discovered experimentally in 1961 (original figure shown in Figure 2). This effect, now called inertial focusing, has received renewed attention in recent years due to its use to arrange and sort single cells in microfluidic devices.

   Let's explore whether this basic effect can be determined via comsol. Again, in the interest of computational time, let's do the problem in 2D. Set up a dimensionless channel of height 2 (from $-1 < y < 1$). Set the length to 20 units. Place the particle (a circle) in the middle of the channel with respect to x. Set the particles diameter

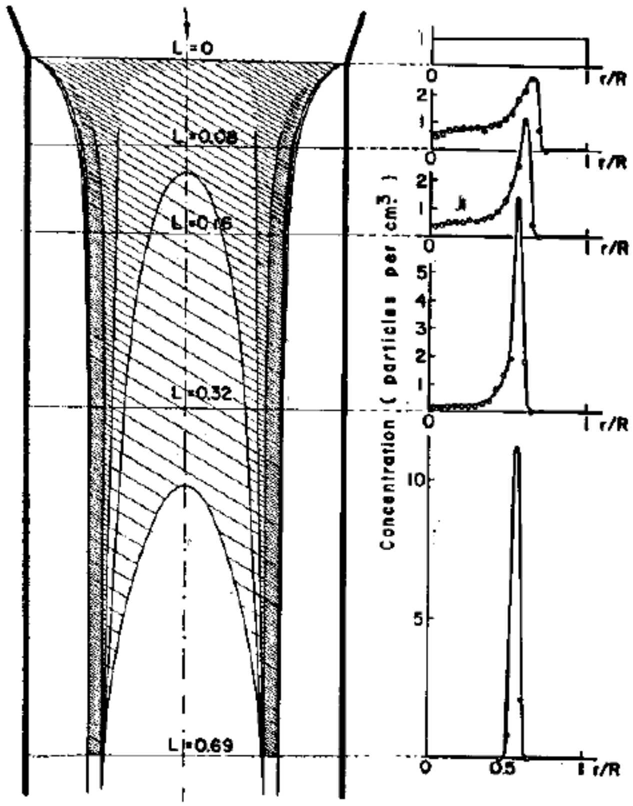

Fig. 2. The 'tubular pinch' effect: particle concentration as a function of radial and longitudinal position in tube. Initial concentration 1 part./cm.³. Concentrations are indicated by shading; closest shading (more than 2 part./cm.³); next (1–2 part./cm.³); next (0·5–1 part./cm.³); lightest shading (0–0·5 part./cm.³)

FIG. 2 Image from Segre and Silberberg, Nature, 1961. In the experiment particles are introduced to a laminar channel flow at uniform concentration. Downstream, the particles focus to a particular radial location.

to 0.1 to start with. Set the inlet velocity field to be $u(y) = 1 - y^2$. Set the outlet to be zero pressure. The upper and lower boundaries should be solid walls with no-slip. Set the viscosity to 0.01 such that the channel Reynolds number is 100 and the particle Reynolds number is 10.

Compute the x and y component of the drag force on the circle. To compute the drag force (we can show you how to do this in class) add "derived values" in results, select line integration, select the walls of the cylinder, click on replace expression, under component 1, laminar flow, auxiliary variables, total stress - select normal stress x-component. Repeat for the y-component. The x-component of the force is the drag and the y-component of the force is the lift.

Now, compute the y component of the drag force as a function of y location of the particle. What does this analysis say the equilibrium position of the particle would be? How is the simulation not exactly the same as a free particle flowing down the channel? What effects are we neglecting?

VI. SOME CALCULATIONS

1. A velocity field for a 2D flow on the domain $-1 < y < 1$ and for all x is found to be $u(y) = (1 - y^2)$ where u is the x-component of the velocity. This flow field for two parallel plates at $y = 1$ and $y = -1$ (here y is a dimensionless height and the velocity field is dimensionless too). Write out the 2D stress tensor. What is the value of the stress vector acting on the plate at $y = -1$? What is the stress vector acting at $y = 0$.

2. Take the so-called Taylor-Green velocity field in 2D

$$u(x, y) = \cos x \sin y$$

$$v(x, y) = -\sin x \cos y$$

Plot the velocity field as a vector field on the domain $0 < x < 2\pi$ and $0 < y < 2\pi$. Compute the stress tensor. Compute the stress vector acting on the surface located at $y = \pi/2$ and $y = \pi$. Plot the x and y component of the vector as a function of x. Look at the flow field and see if you can make some sense of these plots.