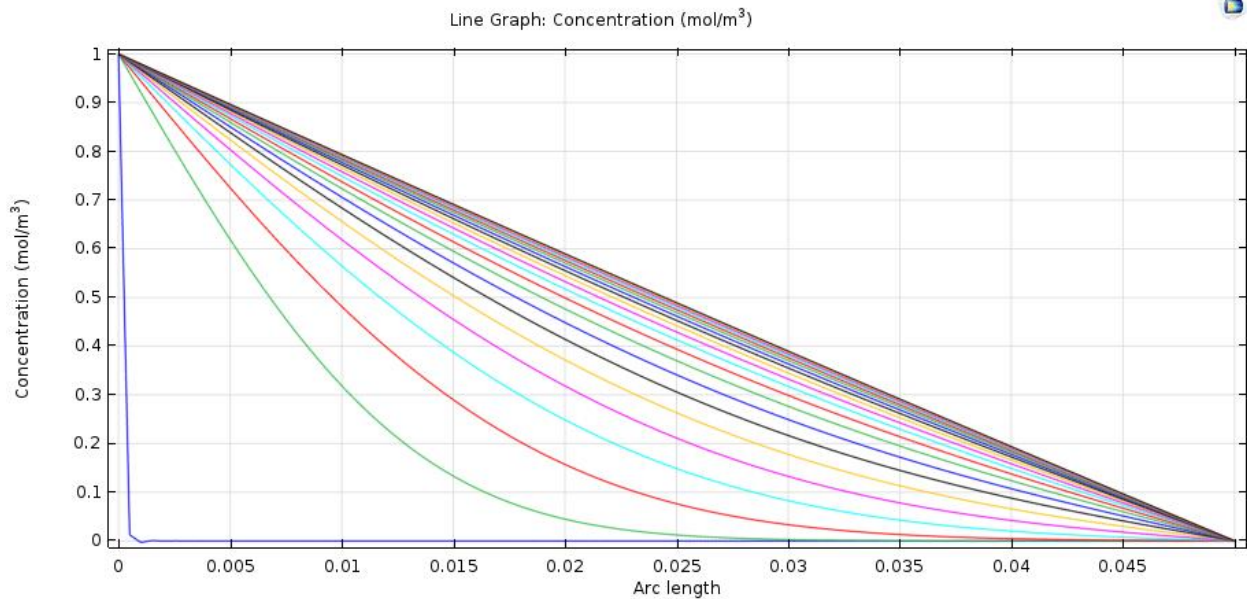


Diffusive mass transfer

Problem 1:



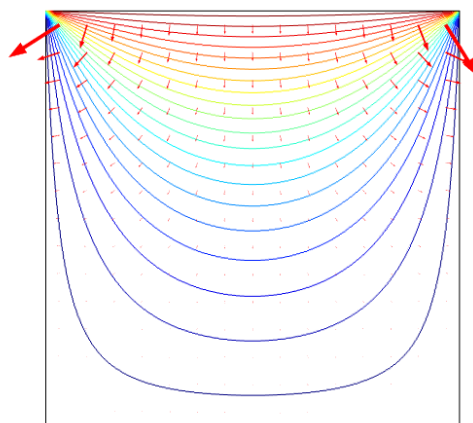
Solution final time is 1,000,000 sec. Time between curves is 50,000 sec.

Time scale using $L^2/D = 2,500,000$. Note that solution is equilibrated before we reach L^2/D , but that the simple estimate gives us the right number of zeros.

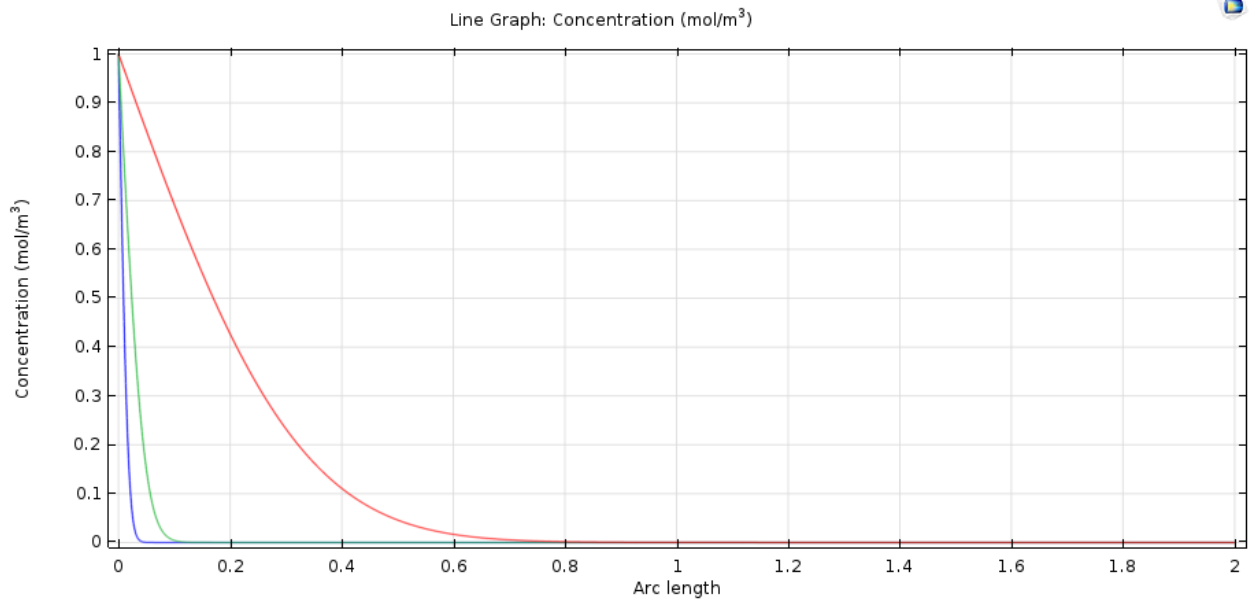
To make the solution dimensionless we can transform the domain to be $0 < x < 1$, and the constant D would become one. In this dimensionless form, we would integrate the equation for 1 unit of time and would expect to see the solution at equilibrium by this time.

Without comsol, we would easily find that the equilibrium state is a line between 0 and 1.

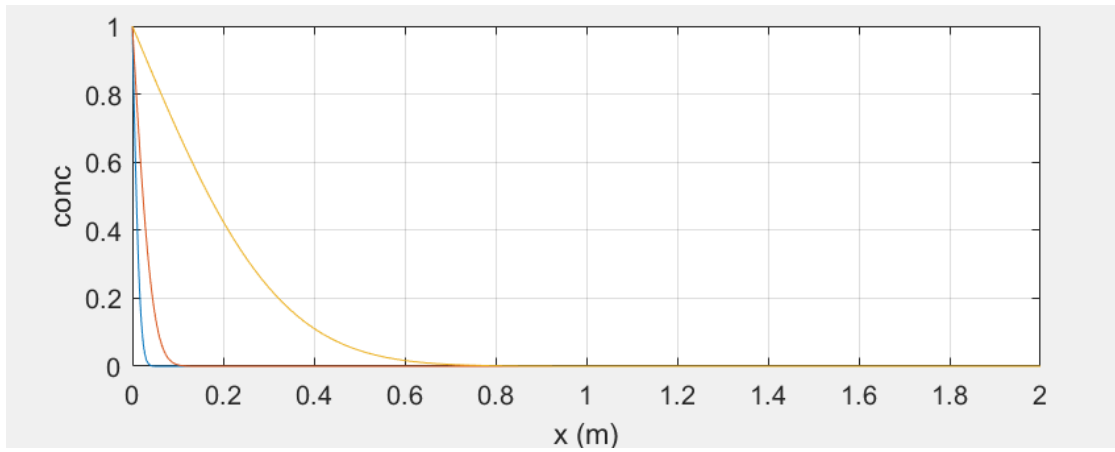
Problem 2:



Problem 3:

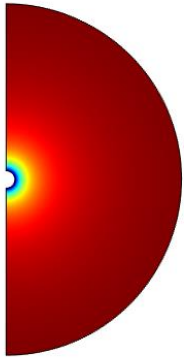


Lines are for 1 day, 1 week, and 1 year. Here is the analytical solution. Comparison looks pretty solid. If we exported the data and overlaid the two plots, they would be an exact match.

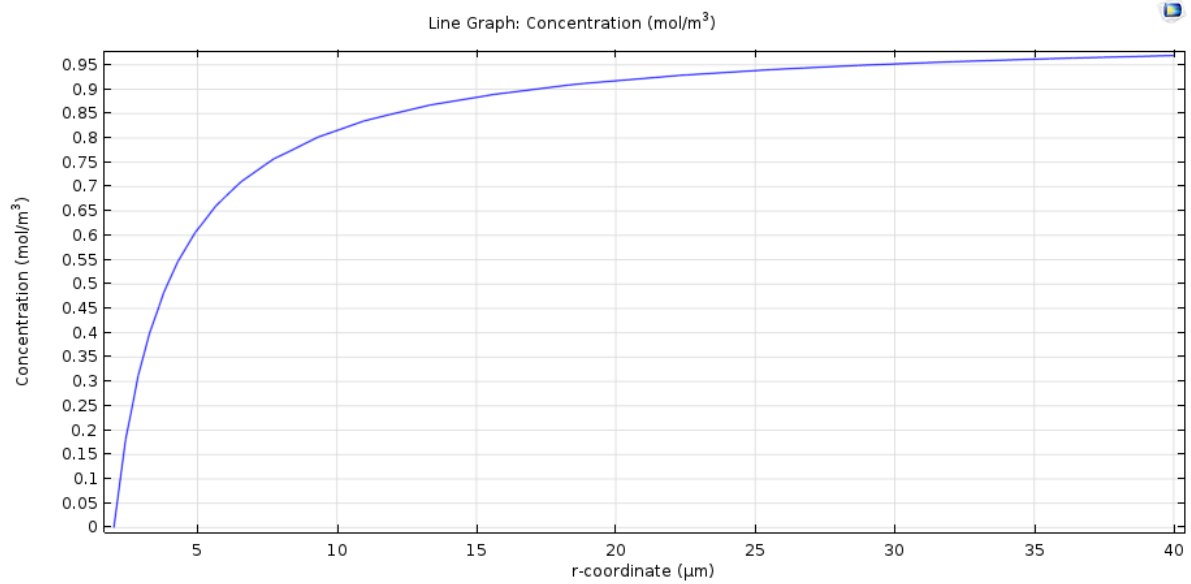


Problem 4:

Image from Comsol.



1D plot from Comsol



Equilibrium state is

$$\nabla^2 c = 0$$

In spherical coordinates where we have axial symmetry this becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) = 0$$

or

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) = 0$$

Integrating once

$$\left(r^2 \frac{\partial c}{\partial r} \right) = C_1$$

$$\frac{\partial c}{\partial r} = \frac{C_1}{r^2}$$

Integrating again

$$c = \frac{-C_1}{r} + C_2$$

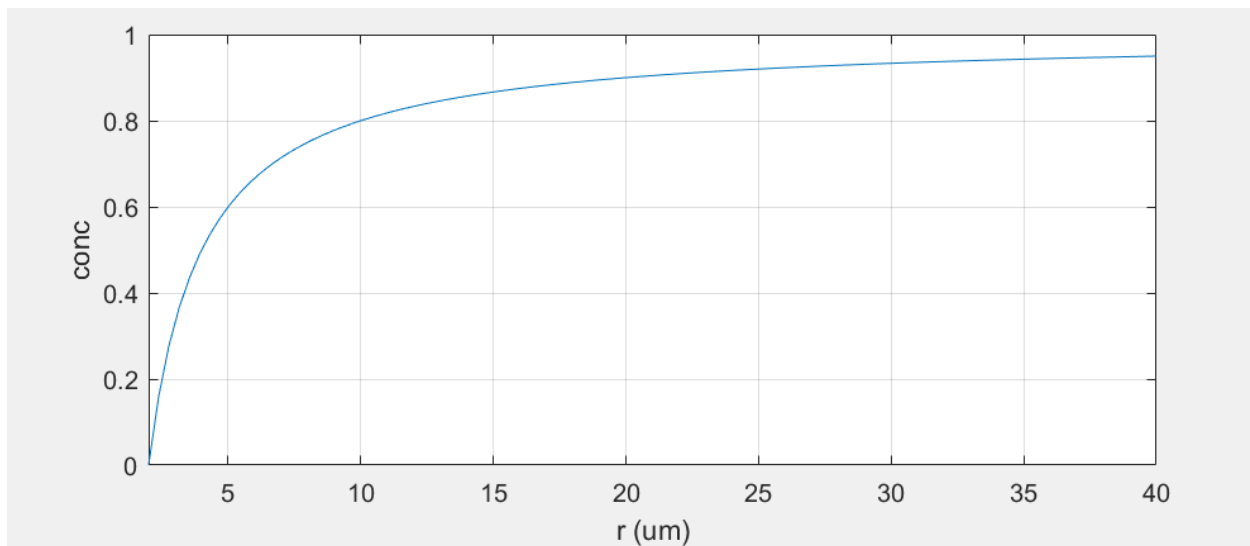
Applying boundary conditions.

$$c(r = \infty) = 1 \text{ therefore } C_2 = 1$$

$$c(r = a) = 1 \text{ therefore } C_1 = a$$

Final result

$$c(r) = 1 - \frac{a}{r^2}$$



Convective mass transfer

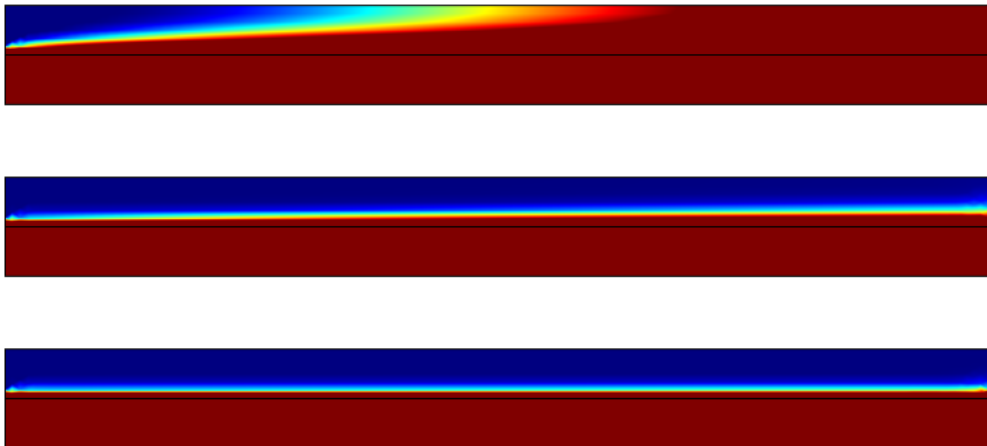
Problem 1:

$D = 0.01, 0.001, \text{ and } 0.0001$ from top to bottom. All taken at $t=20$. Channel is 50 units long.



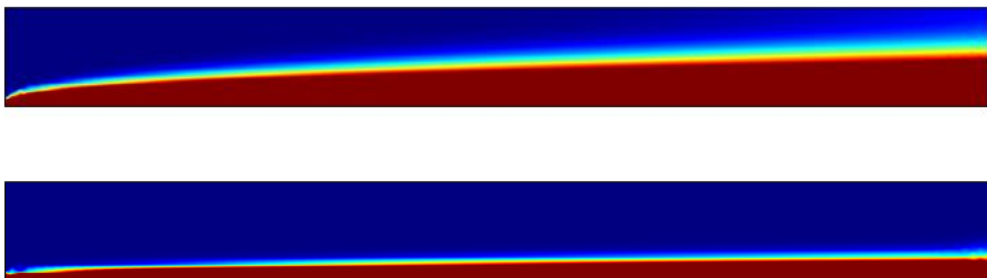
Problem 2:

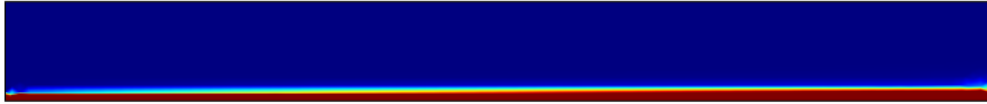
$D=0.01, 0.001, \text{ and } 0.0001$ at steady state



Problem 3:

$D=0.01, 0.001, \text{ and } 0.0001$ at steady state





Problem 4:

All snapshots taken at $t=20$

$D=1$



$D=0.1$



$D=0.01$



$D=0.001$

