#1


```
clear;
figure(1); clf;

N = 10000;
x = zeros(N,1);

Nsteps = 10000
dt = 0.01
t = 0;
X = linspace(-2,2,10000);
D = 0.01;
for i  = 1:Nsteps
    x = x+randn(N,1)*dt;
    hist(x,50);
    axis([-2 2 0 N/10])
    drawnow
end
```

#2


```
clear;
figure(1); clf;

N = 100000;
x = zeros(N,1);
y = zeros(N,1);



Nsteps = 1000
dt = 0.01

t = 0;
for i  = 1:Nsteps

    x = x+randn(N,1)*dt;
    y = y+randn(N,1)*dt;
    r2 = (x.^2+y.^2);
    t = t+dt;

    subplot(2,1,1)
    plot(x,y,'.')
    axis([-2 2 -2 2])


    subplot(2,1,2)
    plot(t,mean(r2),'+')
    hold on
  drawnow
end
```

(3)

Estimates

O_2 $\quad \mathcal{D} \sim 2 \times 10^{-9}\ m^2/s$

$$t \sim L^2/\mathcal{D} \Rightarrow \frac{(5 \times 10^{-6})^2}{2 \times 10^{-9}} = 0.0125\ s$$

$$L^2/\mathcal{D} \Rightarrow \frac{(1 \times 10^{-6})^2}{2 \times 10^{-9}} = 0.5\ ms$$

Sugar

$$L = 5\ cm = 0.05\ m$$

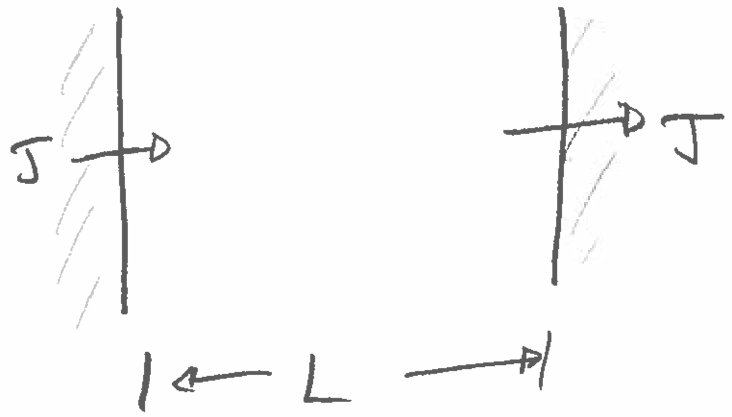
$$t \sim \frac{L^2}{\mathcal{D}} = \frac{(5 \times 10^{-2})^2}{5 \times 10^{-10}} = 57 \text{ days}$$

acetone

$$L \approx 0.5\ m$$

$$t \sim \frac{L^2}{\mathcal{D}} = \frac{(0.5)^2}{1.2 \times 10^{-5}} \approx 5.8 \text{ hrs}$$

(4)



at equilibrium $\frac{\partial^2 c}{\partial x^2} = 0 \Rightarrow c$ is linear

$$J = -D \frac{\partial c}{\partial x}$$

since J is constant + c is linear at steady state.

$$J = -D \frac{dc}{dx}$$

$$c = -\frac{J}{D} x + A$$

A const. of integrati..

$$C(x=0) = C_0$$

$$C = -\frac{J}{D} x + C_0$$

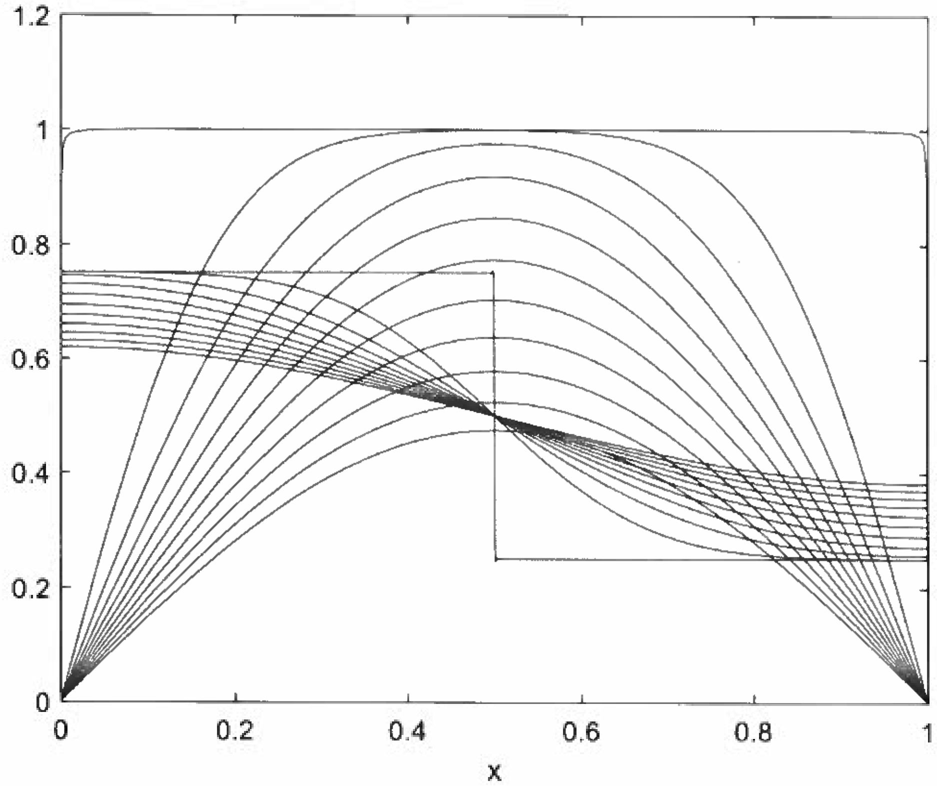
$$\Delta C = \frac{\partial c}{\partial x} L = -\frac{JL}{D} \equiv \frac{mol}{s \cdot m^2} \frac{m}{\frac{m^2}{s}} = \frac{mol}{m^3}$$

#5


```
clear;
x = linspace(0,1,1000)';

for t = 0:0.01:0.1

    c = 1/2*ones(length(x),1);
    for n =1:1000
        c = c + sin(n*pi/2)/(n*pi)*exp(-n^2*pi^2*t)*cos(n*pi*x);
    end
    plot(x,c)
    hold on
end
```


1.2
1
0.8
0.6
0.4
0.2
0
c
0
0.2
0.4
0.6
0.8
1
x

(6)

$$\frac{\partial c}{\partial t} = \mathcal{D} \frac{\partial^2 c}{\partial x^2}$$

$$C(x=0,t)=0$$
$$C(x=L,t)=0$$

$$C(x,t=0)=C_0$$

$$0<x<L$$

to make dimensionless

$$\tilde{x} = x/L \quad , \tilde{t} = t/t_0 \quad , \tilde{c} = c/c_0$$

$$\frac{\partial c_0\tilde{c}}{\partial(\tilde{t}t_0)} = \mathcal{D}\frac{\partial^2 c_0\tilde{c}}{\partial(\tilde{x}L)^2} \Rightarrow \frac{c_0}{t_0}\frac{\partial\tilde{c}}{\partial\tilde{t}} = \mathcal{D}\frac{c_0}{L^2}\frac{\partial^2\tilde{c}}{\partial\tilde{x}^2}$$

$$\frac{\partial\tilde{c}}{\partial\tilde{t}} = \frac{\mathcal{D}t_0}{L^2}\frac{\partial^2\tilde{c}}{\partial\tilde{x}^2} \qquad \text{set } t_0 = L^2/\mathcal{D}$$

$$\boxed{\begin{array}{l} \dfrac{\partial\tilde{c}}{\partial\tilde{t}} = \dfrac{\partial^2\tilde{c}}{\partial\tilde{x}^2} \\ \tilde{C}(\tilde{x}=0,\tilde{t})=0 \qquad \tilde{C}(\tilde{x}=1,\tilde{t})=0 \\ \tilde{C}(\tilde{x},\tilde{t}=0)=1 \\ 0<\tilde{x}<1 \end{array}}$$

Dimensionless problem.

I'm dropping all the ~ now.

$$C = e^{-\lambda^2 t} f(\lambda x)$$

$$\frac{\partial C}{\partial t} = -\lambda^2 e^{-\lambda^2 t} f(\lambda x)$$

$$\frac{\partial C}{\partial x} = e^{-\lambda^2 t} \lambda f'(\lambda x)$$

$$\frac{\partial^2 C}{\partial x^2} = e^{-\lambda^2 t} \lambda^2 f''(\lambda x)$$

$$-\cancel{\lambda^2} \cancel{e^{-\lambda^2 t}} f(\lambda x) = \cancel{\lambda^2} \cancel{e^{-\lambda^2 t}} f''(\lambda x)$$

$$-f(\lambda x) = f''(\lambda x)$$

Fcn that satisfy this:

$$f(\lambda x) = A \sin(\lambda x) + B \cos(\lambda x)$$

So

$$C = e^{-\lambda^2 t} \left(A \sin(\lambda x) + B \cos(\lambda x) \right)$$

Apply BC

$C(x=0)=0$

$$C(0,t) = e^{-\lambda^2 t}\left(A\sin(0) + B\cos(0)\right) = 0$$

$$= e^{-\lambda^2 t} B = 0$$

therefore $B=0$

$c(x=1)=0$

$$C(1,t) = e^{-\lambda^2 t}\left(A\sin\lambda\right) = 0$$

Only way $\sin\lambda = 0$

$$\lambda = n\pi$$

$$C(x,t) = e^{-n^2\pi^2 t} A_n \sin(n\pi x)$$

these satisfy eqn + Both B.C.

therefore any combination.

$$C(x,t) = \sum A_n e^{-n^2\pi^2 t} \sin(n\pi x)$$

Apply initial condition to get A_n

$$C(x,0) = \sum_1 A_n \sin(n\pi x) = 1 \quad \text{(initial condition)}$$

use fact that

$$\int_0^1 \sin(n\pi x)\sin(m\pi x)\,dx = 0 \qquad n \neq m$$
$$= \tfrac{1}{2} \qquad n = m$$

$$\int\left(1 - \sum_1 A_n \sin(n\pi x)\right) \sin(m\pi x)\,dx$$

$$\int_0^1 \sin(m\pi x)\,dx = \int_0^1 A_n \underbrace{\sin(n\pi x)\sin(m\pi x)\,dx}_{\text{all are zero except } n=m}$$

$$\int_0^1 \sin(m\pi x)\,dx = \tfrac{1}{2} A_n$$

$$A_n = 2\int_0^1 \sin(n\pi x)\,dx = \frac{-2}{n\pi} \cos(n\pi x)\Big|_0^1 =$$

$$A_n = \frac{-2}{n\pi}\left(\cos(n\pi) - 1\right)$$

$$A_n = \frac{4}{n\pi} \quad n \text{ is odd}$$

$$A_n = 0 \quad n \text{ is even}$$

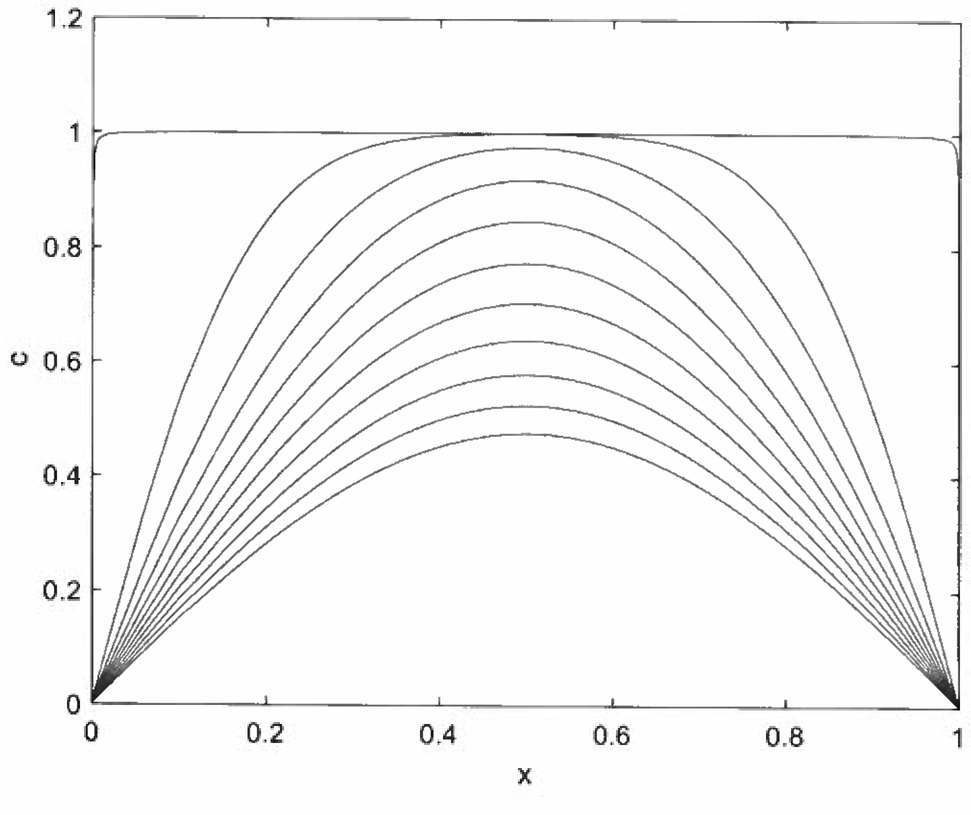
$$C(x,t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi} e^{-n^2\pi^2 t} \sin(n\pi x)$$

Plot Solutions - See Next

```
clear; clf
x = linspace(0,1,1000)';

for t = 0:0.01:0.1

    c = zeros(length(x),1);
    for n =1:2:2000
        c = c + 4/(n*pi)*exp(-n^2*pi^2*t)*sin(n*pi*x);
    end
    plot(x,c)
    hold on
end
xlabel('x'); ylabel('c')
```


1.2
1
0.8
0.6
0.4
0.2
0
c
0
0.2
0.4
0.6
0.8
1
x