Drag and pipe flow

I. READING AND LECTURES

To support this homework

- Read chapter 3 from the book.

II. DRAG ESTIMATES

Recall from Chapter 3, if the flow is dominated by viscosity that dimensional analysis gave a force on a sphere to have a form

$$F = C\mu U D,$$

and if the flow was dominated by inertia the force had a form

$$F = C\rho U^2 D^2.$$

In both cases, the constant C (or Drag Coefficient) is unknown until one conducts additional analysis (we don't know how to do this yet), numerical simulations (we don't know how to do this either), or experiments (which we will do today). Fortunately, for many common shapes drag coefficients have been measured and can be found in books or the internet. When using drag coefficient data you must take care in noting the form of the formula, and what area or length scale is used. Most drag formulas are written assuming inertia dominated flow and use a form

$$F = C_D \frac{1}{2} \rho U^2 A.$$

where A is often the area projected by the object to the flow. Again, the area used in the formula should be reported along with the drag coefficient (often it is not) to avoid any ambiguity. If the drag coefficient is measured but the area that was used to compute C_D is not reported, then the result is useless.

The transition between the inertia dominated flows and viscosity dominated flows is given by the Reynolds number, defined as

$$\mathrm{Re} = \frac{\rho U D}{\mu}.$$

Note that the use of the diameter in the definition of the Reynolds number for flow around a sphere is the accepted convention, but arbitrary. We could just have easily used the radius. Experimental data should always report what length scale is used to define the Reynolds number.

The Reynolds number is usually large for things that you interact with on a daily basis - a 1 mm object traveling at 1 mm/s in water will have a Reynolds number of 1. What counts as high or low Reynolds number depends on the situation, but usually greater than 1000 would be considered well in the high Reynolds number regime.

Let's start this assignment by doing some experiments. In class, we will conduct measurements on spheres falling through a fluid. We will use air, water, and high viscosity silicone oil as our fluids. We will use object of different sizes.

1. Using a variety of objects that we provide, time their fall at terminal velocity (we will provide the objects and some guidance in class). From the measurement of the terminal velocity, the objects size, and the known properties of the fluid we can compute the Reynolds number and the drag coefficient. For each experiment, take a few data points to see how repeatable the experiment is. We will drop objects in air, water, and oil to come up with a handful of data points for drag coefficient versus Reynolds number. Overlay your experimental data on the standard accepted curve (we will provide this empirical correlation). Note that the standard curve uses the projected area of the sphere, πR^2, for the area in the drag formula.

The following problems are estimates - that is important to remember. For each estimate, you can use any resources you can find to get drag coefficients. For each problem, estimate the Reynolds number.

2. How much power is lost to drag on a typical car going 55 mph? 75 mph?

3. During a thunderstorm, hail is created when strong updrafts can keep large pieces of ice aloft. When the ice grows to a mass too great, the wind can't hold it up and the ice falls down as hail. How strong are the updrafts needed to create golf ball size hail?

III. FLOW IN PIPES

Through dimensional analysis, we can figure out the general form of the pressure drop needed to force fluid through a tube . The pressure drop is usually written as

$$\Delta P = \frac{1}{2}\rho U^2 f \frac{L}{D}$$

where f is the friction factor and is a function of the Reynolds number. When the Reynolds number is less than about 2300, the flow is smooth and laminar and the friction factor can be found analytically (later in this course, we will compute this solution)

$$f = \frac{64}{Re} \quad \text{for Re} < 2000.$$

The Reynolds number is defined using the tube diameter as the appropriate length scale and uses the average velocity, U. When the Reynolds number is larger than about 200 the flow becomes turbulent and complex and f is found by use of the Moody chart which is based on experimentally measured data. For turbulent flows, f is a function of the Reynolds number and the roughness of the pipe.

In class we will conduct a number of experiments on flow through tubes. We will use smooth flexible plastic tubing.

1. Collect data as volumetric flow rate as a function of water height in the driving reservoir. Remember that it is the height difference between the top of the tank and the exit of the tube. To get good data, try to keep the tube as straight as possible - curves and bends add extra resistance. Also, it helps at low flow rates to discharge the pipe under the surface in a pool of liquid - not allowing the tube to drip. If the tube is dripping, then the water's surface tension is resisting the flow and interfering with your results.

   Convert height to driving pressure as $P = \rho g h$. Convert volumetric flow rate to velocity as $U = Q/(\pi R^2)$. Using the pressure and velocity, compute the Reynolds number and friction factor. We will provide a few tube diameters and provide some guidance as a reasonable parameter range to cover. For each diameter tube try a few water heights to get a range of Reynolds number. Overlay your $f(Re)$ data points on the standard accepted Moody diagram for smooth pipes.

Now try a few estimates for flow in a pipe,

5. How much pressure is needed to deliver water through a 1/2 inch diameter pipe (typical in home plumbing) at 1 gallon per minute over a length of 50 ft of pipe. Convert your pressure to equivalent height - i.e. hydrostatic pressure is given as $\rho g h$. Note that you will need to check if the Reynolds number of this flow puts you in the laminar or turbulent regime. If $Re > 2300$ then DO NOT use $f = 64/Re$.

6. Typical water pressure supplied to a home is around 50 psi. At this inlet pressure, what would be the flow rate through 50 feet of 1/2 inch diameter pipe, assuming no other losses? Note that this problem is a little different than when I tell you the flow rate since now you don't know the velocity a priori to compute Re and thus f. You will need to iterate - guess f, compute the velocity, check f on the Moody diagram for that Reynolds number, get f on the Moody chart for the Re, recompute.

7. In various applications of microfluidics we often need to force relatively high flow rates through narrow channels. Taking a tube to be 40 microns in diameter and 30 mm in length, what is the required pressure to deliver 10 ml/hour of water (a high flow rate for such a small channel). If the flow is delivered via a standard 10 ml syringe with a diameter of 14.5 mm, what force is applied to the syringe?