Dimensional analysis

I. READING AND LECTURES

To support this homework, you should have already completed the following.

- Read chapters 1 and 2 from the book.
- Watch "Dimensional analysis" video on the youTube playlist.
- Read "Principle of similitude", by Lord Rayleigh
- Watch this movie by G.I. Taylor. It's awesome. "Low-Reynolds-Number Flows"
  https://youtu.be/51-6QCJTAjU

II. PI THEOREM

1. Derive with the table method and Pi Theorem, Rayleigh's statement that "The velocity of propagation of periodic waves on the surface of deep water is as the square root of the wavelength." State what you had to assume/know to get this answer.

2. A famous problem in dimensional analysis was when G.I. Taylor calculated the energy released during the first atomic bomb tests (a classified secret) from unclassified images that were released to the public. His argument was based on dimensional analysis. His physical insight was that the radius of the blast r (which he could measure from the images), was only a function of time t (which he also new from the images), the density of the surrounding air ρ and the energy released E. Thus, $r = f(t, E, \rho)$. Express this functional dependence in dimensionless form.

3. Derive with the table method and Pi Theorem, the form for the drag force on a sphere. Imagine a sphere of diameter, D, traveling at constant velocity U, in fluid with density ρ and viscosity μ. (Note that μ is the dynamic viscosity and has units of Pascal-seconds or [M]/[L T]). Express the drag force, F, as a function of the other variables.

   There are two important limits in this problem that we will discuss in further detail next week.

   - For large objects at high speeds (i.e. a skydiver, an airplane, a baseball), the fluid inertia (i.e. the mass density) dominates the drag force and viscosity is experimentally observed not to influence the drag force. In this limit the drag force only depends on ρ, D, and U.
   - Small objects very low speeds (i.e. a small organism, a speck of dust) the fluids inertia is unimportant and thus its mass density is observed not to influence the drag force. In this limit the drag force only depends on μ, D, and U.

   Derive the form of the drag force on the sphere in these two limits. If these limits don't not make physical sense to you yet, that is OK, we will discuss this at length later in the course.

4. Take an index card. Cut it in a square shape. Hold it with the flat side facing the ground and drop it. Time the descent with a stop watch. Take a couple of timings - there will be some variation as the card will flutter to the ground differently every time. Make sure you use a consistent fall height (and that you know it) for each measurement. If you bend the card a little bit upward you can get it to fall reasonably steady. You can also try to tape two cards together to make it a little thicker and it will not flutter as much. Cut the square so it has about half the side length of the original. Repeat the experiment. Make another square of half the size again and repeat the experiment. You should now have a data set of fall times for three different sized squares where the thickness is constant.

   Now take one size square. Tape two cards together to double the thickness. Time the fall. Finally, quadruple the original thickness and time the fall. You should now have a data set which varies thickness while holding square side length constant.

   Use the drag law derived in the last problem for the limit where viscosity does not matter to explain your experimental findings. Assume the card is at terminal velocity and thus the force of gravity balances the drag force. Does the prediction from dimensional analysis explain your experimental findings?

Finally, drag coefficient (which we will discuss more next week) in this case would be defined as

$$C_D = \frac{F}{\frac{1}{2}\rho U^2 A}$$

where A is the area of the square, U is the velocity, and ρ is the density of air. Using this equation and the fact that $F = mg$ at terminal velocity, compute the coefficient of drag for each experiment. Comment on whether (and how) the drag coefficient seems to vary with square size and card thickness. Can a single drag coefficient reasonably explain your experimental data?

5. In the book, the problem of pressure driven flow through a pipe is addressed from a dimensional analysis point of view. For low flow rates in small tubes (we will discuss next week more what we mean by "low" and "small") the fluid density is not important to the relationship between pressure and flow. The reason is connected to problem 3 which discusses regimes for drag where density is unimportant - again we will be exploring these ideas in the coming weeks further.

   Rework the dimensional analysis table from page 21 in the book, but just assume that density is unimportant and is not a parameter. From a dimensional analysis perspective, derive the relationship (up to a constant) between applied pressure and volumetric flow rate. For a fixed pressure difference how does the flow rate scale scale with pipe diameter?

   We can test this prediction experimentally. We will conduct an experiment with 1/16 and 1/32 inch tubing in class. Also see the G.I. Taylor movie and watch his version around time 5:40.