Solutions to Navier-Stokes

In the following problem we will make use of the simple solutions to the Navier Stokes where we assume long channels where the flow doesn't change along the length of the channel. In this case, the flow velocity only changes across the channel. In all the problems here we construct the solution as the sum of Couette and Poiseuille flow.

We will demonstrate a simple experiment in class of a solid cylinder dropping through a liquid filled tube. Outside of class, I recorded several data points which are reported in Table 1. In the experiment we can vary different parameters such as the fluid and the geometry.

D_{cyl} (mm)	L_{cyl} (mm)	M_{cyl} (g)	D_{tube} (mm)	μ (Pa-s)	ρ (kg/m^3)	V (cm/s)
24.43	100	66.4	25.8	0.0009	1000	2.05
23.34	100	61.2	25.8	0.0009	1000	7.4
22.43	100	56.1	25.8	0.0009	1000	14.2
24.38	200	132.5	25.8	0.0009	1000	2.05
24.43	100	66.4	25.8	0.0495	1210	0.042
23.34	100	61.2	25.8	0.0495	1210	0.177
22.43	100	56.1	25.8	0.0495	1210	0.412

TABLE I Experimental data for a cylinder dropping in a tube.

I would like you to observe the experiment, and try to explain the behavior you see. This exercise is closer to how a "real" problem is stated rather than the usual book problems. A few things to guide you, though ulitmately all I am after is a simple expression that explains the overall behavior and a Simulation of the full problem.

- Start with a 2D approximation - i.e a thin gap. This assumption will speed up and simplfy your ability to compute things with comsol. Set up a 2D analog of the problem in Comsol. It is easiest to start by assuming you have a problem where you can fix the speed of the shuttle and you are trying to compute the total force on the object.
- Once you have a Comsol simulation, work out the 2D problem analytically assuming a sum of Poiseuille and Couette flow. You can now compare Comsol results to your analysis on a one to one basis to make sure it is correct.
- Using your 2D analysis, assume a very thin gap and come up with an analytical relationship that explains the data. In assuming a thin gap, you ignore the curvature of the gap and just multiply the stress in the gap by $2\pi R$ to get the total viscous stress. You should be able to get a simple expression that fits all the data pretty well.
- Conduct a fully 3D simulation in Comsol and compare the comsol results to both the 2D model and the experimental data.