Fluid motion and mass transfer

I. VIDEO LECTURES

This week you should watch

- Videos for chapters 5, http://fluids.olin.edu/Lectures.shtml

II. READING

This week you should read

- Chapter 5.

III. READING QUESTIONS

1. Write out, in component form, Dc/Dt, where c would be the scalar concentration field.
2. What is Fick's Law?
3. Explain, in words what the material derivative represents.
4. Imagine I had a book full of solutions to the 1D heat equation for different initial and boundary conditions. How could I use this book to solve problems of *diffusive* mass transfer (without fluid motion).

IV. DIFFUSIVE MASS TRANSFER (NO FLUID MOTION)

1. Write a short MATLAB (or python or whatever) program to simulate the 1D random walk of a set of particles. Locate 10,000 particles at the origin $x = 0$. At each time step move the particles a distance proportional to a normally distributed random number (use the `randn` command). Use the MATLAB vector notation to keep this simple. For example to update the particle position at each time step you can simply do `x = x + dt*randn(size(x));` where dt is a time step size that doesn't matter - just a scaling factor. Use the `hist` command to plot the distribution of particles. Compare the result to the analytical solution based on the diffusion equation which is

$$c(x,t) = \frac{C}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

   where $C = \int_{-\infty}^{\infty} c(x,t)dx$ is the total amount of stuff. While a one to one comparison is possible, you can just confirm the result qualitatively. If your program is more than a few lines long, see me for some MATLAB help.

2. These are estimates to get a sense for some orders of magnitude.
   - The diffusivity of dissolved oxygen in blood is about $2 \times 10^{-9} \mathrm{m}^2/\mathrm{s}$. How long does it take oxygen to diffuse over a distance of 5 microns (the scale of capillaries) ? 1 micron (the barrier between oxygen and blood in the lungs)?
   - Typical diffusion constants for sugar in water are $5 \times 10^{-10} \mathrm{m}^2/\mathrm{s}$. How long would it take for diffusion to mix sugar in a cup of water.
   - Typical diffusion constants for acetone in air are around $1.2 \times 10^{-5} \mathrm{m}^2/\mathrm{s}$. How long would it take you to smell a bottle after opening if there were only molecular diffusion.

3. Imagine a 1D problem where a stationary fluid is bounded by two walls seperated by a distance L. On the left wall, there is a chemical reaction at the wall that produces chemical i at a rate of J $\mathrm{mol}/(\mathrm{s} \cdot \mathrm{m}^2)$. On the right there is a chemical reaction that consumes chemical species i. The diffusivity of the chemical species in the fluid is D. At steady state the consumption rate must equal the production rate. At steady state, what is the difference in concentration across the two walls in terms of the parameters provided?

V. CONVECTIVE MASS TRANSFER

Use comsol for the following problems. We will do all these problems in normalized, or dimensionless form. Select at the beginning of the problem setup, “Chemical species transport”, “Transport of diluted species”. Set up a 2D rectangle to be 2 units high (in the y-direction) by 20 units long (in x-direction). The vertical dimension of the domain should go from -1 to 1. You can just leave comsol in units of m and we will control the important dimensionless parameter via changing the diffusivity by hand (rather than using a built in material property). Once you have the geometry specified, under “transport of diuted species” on the left, select “transport properties”. Set the velocity field to be a parabola, $u(y) = 1 - y^2, \;\; v = 0$, where u and v are the x-y components of velocity respectively. We will soon learn why the parabolic velocity profile holds when we have flow between two plates. Set the diffusivity to be 0.01 initially, but you will change this parameter. Solve the following problems.

1. Solve a transeint problem. Set the $y = 1$ and $y = -1$ boundaries to be no flux (the default). Set the $x = 20$ boundary to be outflow. Set the $x = 0$ boundary to be $c = 1$. Change the integration time to be 50. Run the simulation and watch the animation. After running with $D = 0.01$, change $D = 0.001$, then 0.00001. Observe the behavior.

2. Solve a transeint problem. Set the $y = 1$ and $y = -1$ boundaries to be no flux (the default). Set the $x = 20$ boundary to be outflow. Set the $x = 0$ boundary between $0 < y < 1$ to be $c = 1$. Set the $x = 0$ boundary between $-1 < y < 0$ to be $c = 0$. Change the integration time to be 50. Run the simulation and watch the animation. After running with $D = 0.01$, change $D = 0.001$, then 0.00001. Observe the behavior.

3. Solve a steady state problem. Set the $y = 1$ boundary to be no flux (the default). Set the $x = 20$ boundary to be outflow. Set the $x = 0$ to be $c = 0$. Set the $y = -1$ to be $c = 1$. After running with $D = 0.01$, change $D = 0.001$, then 0.00001. Observe the behavior.