HW 2: Drag and pipe flow

I. READING AND LECTURES

To support this homework

- Re-read chapter 2 from the book.

II. DRAG ESTIMATES

Recall from Chapter 2, if the flow is dominated by viscosity that dimensional analysis gave a force on a sphere to have a form

$$F = C\mu U D,$$

and if the flow was dominated by inertia the force had a form

$$F = C\rho U^2 D^2.$$

In both cases, the constant C (or Drag Coefficient) is unknown until one conducts additional analysis (we don't know how to do this yet), numerical simulations (we don't know how to do this either), or experiments. Fortunately, for many common shapes drag coefficients have been measured and can be found in books or the internet. When using drag coefficient data you must take care in noting the form of the formula, and what area or length scale is used. Most drag formulas are written assuming inertia dominated flow and use a form

$$F = C_D \frac{1}{2}\rho U^2 A.$$

where A is often the area projected by the object to the flow. Again, the area used in the formula should be reported along with the drag coefficient (often it is not) to avoid any ambiguity. If the drag coefficient is measured but the area that was used to compute C_D is not reported, then the result is useless.

The transition between the inertia dominated flows and viscosity dominated flows is given by the Reynolds number, defined as

$$\mathrm{Re} = \frac{\rho U D}{\mu}.$$

The Reynolds number is usually large for things that you interact with on a daily basis - a 1 mm object traveling at 1 mm/s in water will have a Reynolds number of 1. What counts as high or low Reynolds number depends on the situation, but usually greater than 1000 would be considered well in the high Reynolds number regime.

In all the problems of this section, state your assumptions. The following are estimates only - that is important to remember. For each estimate, you can use any resources you can find to get drag coefficients. For each problem, estimate the Reynolds number.

1. How much power is lost to drag on a typical car going 55 mph? 75 mph?

2. During a hurricane with 130 mph winds, what is the force exerted on a stop sign?

3. During a thunderstorm, hail is created when strong updrafts can keep large pieces of ice aloft. When the ice grows to a mass too great, the wind can't hold it up and the ice falls down as hail. How strong are the updrafts needed to create golf ball size hail?

4. A sperm cell about 4 microns in diameter (assume a sphere) can propel itself at about 50 microns per second. What is the power density (power per unit volume) that this cell needs to maintain this motion? The cell can complete its trip in about 100 minutes - what is the energy density (note that a battery might have an energy density of 100 Watt-hours/Liter)? To help with the drag calculation - Google "Stoke's law".

III. FLOW IN PIPES

Through dimensional analysis, we can figure out the general form of the pressure drop needed to force fluid through a tube. The pressure drop is usually written as

$$\Delta P = \frac{1}{2}\rho U^2 f \frac{L}{D}$$

where f is the friction factor and is a function of the Reynolds number. Note that this functional form is equivalent (though it may not appear at first glance) as that in Chapter 2. When the Reynolds number is less than about 2000, the flow is smooth and laminar and the friction factor can be found analytically (later is this course)

$$f = \frac{64}{Re} \quad \text{for Re} < 2000.$$

The Reynolds number is defined using the tube diameter as the appropriate length scale and uses the average velocity, U. When the Reynolds number is larger than 2000 the flow becomes turbulent and complex and f is found by use of the Moody chart which is based on experimentally measured data. For turbulent flows, f is a function of the Reynolds number and the roughness of the pipe.

1. Show that the functional form given above for laminar flow (i.e. $f = 64/Re$) is equivalent to the often used Poiseuille's law

   $$\Delta P = QR$$

   where Q is the volumetric flow rate and R is the hydraulic resistance given by

   $$R = \frac{128\mu L}{\pi D^4}.$$

   Note that this law only works for Reynolds number less than $\sim$ 2000. "Derive" the form of Poiseuille's law using dimensional analysis; recall dimensional analysis cannot tell you the factors of 128 and π but it can give you the dependence on length, diameter and viscosity.

2. How much pressure is needed to deliver water through a 1/2 inch diameter pipe (typical in home plumbing) at 1 gallon per minute over a length of 50 ft of pipe. Convert your pressure to equivalent height - i.e. hydrostatic pressure is given as $\rho g h$. Note that you will need to check if the Reynolds number of this flow puts you in the laminar or turbulent regime.

3. Now assume the same problem above but that the pipe made 6 right angles. How much extra pressure drop do the bends induce? Not for this part you will need to look up losses typical in 90 degree bends. Thee losses due to bends in the pipe are called minor losses - if you are looking for the right term to search for.

4. Typical water pressure supplied to a home is around 50 psi. At this inlet pressure, what would be the flow rate through 50 feet of 1/2 inch diameter pipe, assuming no other losses? Note that this problem is a little different than when I tell you the flow rate since now you don't know the velocity a priori to compute Re and thus f. You will need to iterate - guess f, compute the velocity, check f on the Moody diagram for that Reynolds number, get f on the Moody chart for the Re, recompute.

5. In various applications of microfluidics we often need to force relatively high flow rates through narrow channels. Taking a channel to be 40 microns in diameter and 30 mm in length, what is the require pressure to deliver 10 ml/hour of water (a high flow rate for such a small channel). If the flow is delivered via a standard 10 ml syringe with a diameter of 14.5 mm, what force is applied to the syringe? Note that in applications, though small channels are usually made square the formula for a circular tube of the same size works well enough for estimation purposes.

6. Human capillaries are about 6 microns in diameter. Average flow velocities in capillaries are about 1 mm/s. What is the pressure drop across a 1 mm long capillary? Compare this drop to typical blood pressure. For now, you can ignore the extra resistance due to the red blood cells and just assume that the plasma is close to water. The heart pumps about 6 liters per minute. Based on this total flow rate and the fact that capillary flow rates are about 1 mm/s, estimate how many parallel capillaries there are. Note that the main pulmonary artery has an internal diameter about 2.5 cm - compare the total cross sectional area of all the capillaries to that of the primary artery.