HW 1: Dimensional analysis

I. READING AND LECTURES

To support this homework, you should have already completed the following.

- Read chapters 1 and 2 from the book.
- Watch "Dimensional analysis" video on the youTube playlist.
- Read "Principle of similitude", by Lord Rayleigh
- Watch this movie by G.I. Taylor. It's awesome.
  "Low-Reynolds-Number Flows" https://www.youtube.com/watch?v=51-6QCJTAjU

II. PI THEOREM

- Derive with the table method and Pi Theorem, Rayleigh's statement that "The velocity of propagation of periodic waves on the surface of deep water is as the square root of the wavelength." State what you had to assume/know to get this answer.
- A famous problem in dimensional analysis was when G.I. Taylor, the biggest name in fluid dynamics for the 20th century, calculated the energy released during the first atomic bomb tests (a classified secret) from unclassified images that were released to the public. His argument was based on dimensional analysis. His physical insight was that the radius of the blast r (which he could measure from the images), was only a function of time t (which he also new from the images), the density of the surrounding air ρ and the energy released E. Thus, $r = f(t, E, \rho)$. Express this functional dependence in dimensionless form.
- Derive with the table method and Pi Theorem, the form for the drag force on a sphere. Imagine a sphere of diameter, D, traveling at constant velocity U, in fluid with density ρ and viscosity μ. (Note that μ is the dynamic viscosity and has units of Pascal-seconds or [M]/[L T]). Express the drag force, F, as a function of the other variables.

  There are two important limits in this problem that we will discuss in further detail as the course goes on.

  - For large objects at high speeds (i.e. a skydiver, an airplane, a baseball), the fluid inertia (i.e. the mass density) dominates the drag force and viscosity is experimentally observed not to influence the drag force. In this limit the drag force only depends on ρ, D, and U.
  - Small objects very low speeds (i.e. a small organism, a speck of dust) the fluids inertia is unimportant and thus its mass density is observed not to influence the drag force. In this limit the drag force only depends on μ, D, and U.

  Derive the form of the drag force on the sphere in these two limits. If these limits don't not make physical sense to you yet, that is OK, we will discuss this at length later in the course.
- Take an index card. Cut it in a square shape. Hold it straight out arms length with the flat side facing the ground and drop it. Time the descent with a stop watch. Take a couple of measurements - there will be some variation as the card will flutter to the ground differently every time. If you bend the card a little bit upward you can get it to fall reasonably steady. You can also try to tape two cards together to make it a little thicker and it will not flutter as much. Cut the square so it has about half the side length of the original. Repeat the experiment. Make another square of half the size again and repeat the experiment. Use the drag law derived in the last problem for the limit where viscosity does not matter to explain your experimental findings. Assume the card is mostly at terminal velocity and thus the force of gravity balances the drag force.
- Consider a steel sphere of diameter D dropped in a very viscous fluid. Assume that we are in the regime where viscosity dominates. The sphere is dropped in the fluid, comes to terminal velocity very quickly, then descends down a container of a certain depth. If you double the diameter of the sphere, how does the time to reach the bottom change?

III. GEOMETRIC SIMILARITY

In fluid dynamics, if an object is scaled such that it is geometrically similar, a model can be used to extract information about a real system. This is useful because instead of constructing a real experiment on a bridge, we can build a model and test the wind loads in a small wind tunnel, for example. The mass of a geometrically similar model scales as L^3 where L is characteristic length. Imagine a cylinder who's diameter is 1/10 the length. The volume of the cylinder is $V = \frac{\pi}{4}\left(\frac{L}{10}\right)^2 L$; i.e. $V \sim L^3$ where the symbol $\sim$ means scales as. In determining scaling laws, we don't care about the constants just how things scale. For the scaling law, we only care that the volume goes up as the cube of the length. If I double the length the volume goes up by a factor of 8. Even for a more complicated object, if the model is geometrically similar, then there is only one characteristic length for the overall size of the object and the volume or mass scales as the cube of the length.

Now some problems:

- Consider the problem of lift for an airplane and other flying things such as birds and bugs. The lift force F is found experimentally to depend on the velocity U, the fluid density ρ, and the area of the wing A. As with the drag problem, if the flow speed is high enough then the fluid viscosity does not matter. Using the Pi theorem, derive an expression for the lift force as a function of the other parameters. When the object is flying at constant speed the lift force must equal the weight.

  If we now approximate all winged objects of some characteristic size (say their length) L as geometrically self-similar, then we can express the surface area A and weight W as proportional to particular powers of L (see above). Write down these power-law relationships and use them to develop a scaling relationship between the weight W and the cruising velocity U alone. Notice how strong your scaling is (i.e. how big the exponent is).

  Verify that your scaling is correct by plotting the following data (in Matlab) on a Log-Log plot. Note the units and convert to a consistent set of units. Plot a line with the power predicted by your simple analysis. Recall that on log-log plots a function of the form $y = x^m$ is a straight line with a slope of m.

Object	Mass	Wing Area A [m2]	Cruising Speed U
Crane fly	30 mg	7.5e-5	3 m/s
Common starling	80 g	0.02	10.3 m/s
Canadian goose	5.7 kg	0.28	23 m/s
Cessna Citation	2.0 metric tons	18.2	120 mph
Boeing 747	350 metric ton	511.0	570 mph

IV. PUDDLES

A very viscous fluid is poured out on a flat table. Over time the liquid spreads out into a circular puddle. We can take a movie of the process and record the radius of the puddle as a function of time. This was done for three different initial volumes of the same fluid. The important parameters in the problem are the radius, r, time t, density ρ, initial volume V, viscosity μ, and acceleration due to gravity g. The Pi theorem would say that there are 3 parameters in the problem.

However, we can do better with a little simple reasoning - without actually doing the hard work of solving the problem in any exact sense. Since the spreading is so slow, we are in a regime where the fluids inertia (or mass density) is not important for the dynamics. However, the density of the fluid is important in setting the gravitational force which is what causes the fluid to spread. Thus, we assume that the density of the fluid only enters the problem through its combination with g. The "proper" parameter to include is then ρg - not these variables independently. With the assumption of ρg as the parameter, then the Pi theorem says there are only 2 dimensionless parameters that matter.

Take the data from the website. When you import the data into MATLAB, radius is in cm, time in seconds, and volume in mL. Plot r vs. t on log-log coordinates for all experiments on one plot. Plot the 2 dimensionless parameters for all experiments on one plot and see if the data collapse.

While this experiment might seem completely irrelevant, it turns out that this viscous spreading has been observed magma spreading of lava domes. The puddle experiment is a form of a gravity current - flows driven by density differences. Gravity currents are ubiquitous in ocean and atmospheric flows and are critical to understanding weather and climate.