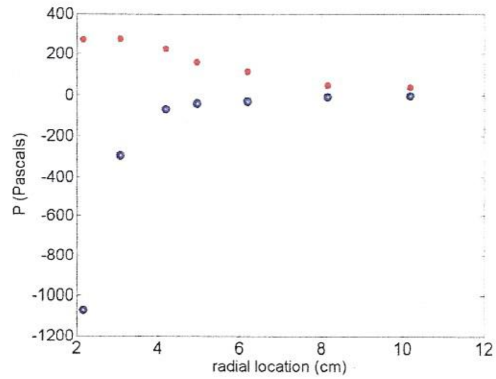


FIG. 1 Pressure data for the levitating plate. The blue data with open circles the device is oriented downward such that the plate is held up with respect to gravity. If the air flow is turned off the plate falls to the ground. For this case the air flow was measured to be 0.0138 m^3/s and the gap height was measured to be 2 mm. The red data with closed dots the device is inverted and the plate is held up with respect to gravity. If the air flow is turned off the plate collapses to the blower. In this case the flow rate was measured to be 0.0023 m^3/s and the gap height was determined to be 0.7 mm.

In class I will demonstrate a simple experimental setup. Essentially air blows out of an opening in a flat plate. Another plate is brought close to the opening and this plate is pulled upward and levitates like an upside down air hockey puck, even though the air is blowing outward. The behavior seems counterintuitive, but is easily explained with basic fluid dynamics.

To make progress, we will simplify the Navier Stokes equations significantly. Start with the Navier-Stokes equations for a Newtonian incompressible flow in cylindrical coordinates. Assume flow is only radial, v_r, such that the theta and z components of the velocity are zero. Also assume the problem is symmetric and all theta derivatives drop out.

- The Navier Stokes in cylindrical coordinates are listed below. Cross out all the terms that are zero by the assumptions above.
- For the case where a constant volumetric flow rate is forced into the inner hole, solve for the velocity as a function of radial coordinate.
- Assume viscosity is zero and remove those terms from your equation.
- Use the momentum equation without viscosity to solve for pressure as a function of radial coordinate.
- Compare your result to the experimental measurements seen in the figure below.
- Your model cannot explain the data where the system is inverted and the plate is on top. Why?

Steady　　　　axisym

Cylindrical Coordinates (r, θ, z):

$$\cancel{\frac{\partial \rho}{\partial t}} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \cancel{\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)} + \cancel{\frac{\partial}{\partial z}(\rho v_z)} = 0$$

No vertical flow

Steady　Sym　Sym　No vertical flow　Sym　Sym

Cylindrical Coordinates (r, θ, z):

$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}} + v_r\frac{\partial v_r}{\partial r} + \cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta^2}{r}} + \cancel{v_z\frac{\partial v_r}{\partial z}}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}} + \frac{\partial^2 v_r}{\partial z^2} - \cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}}\right] - \frac{\partial p}{\partial r} + \cancel{\rho g_r}$$

No gravity in radial direction

$$\cancel{\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta}$$

No Swirl

$$\cancel{\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z}$$

No vertical flow

Left with

Mass:

$$\frac{\partial}{\partial r}(\rho r v_r) = 0$$

$$\rho r v_r = \text{constant.}$$

total flow $Q = v_r \, r \, H \, 2\pi$

gap

$$v_r = \frac{Q}{2\pi r H}$$

Momentum

$$\rho v_r \frac{\partial v_r}{\partial r} = \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right) - \frac{\partial P}{\partial r}$$

Note: Cons. of mass says this term is zero

$$\rho \frac{\partial}{\partial r}\left(\tfrac{1}{2} v_r^2\right) + \frac{\partial P}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2}$$

$$\frac{\partial}{\partial r}\left(\tfrac{1}{2}\rho v_r^2 + P\right) = \mu \frac{\partial^2 v_r}{\partial z^2}$$

if we neglect viscosity

$$\frac{1}{2}\rho V_r^2 + P = \text{Constant}$$

if R is outer radius and $P = P_\infty$ at outer radius.

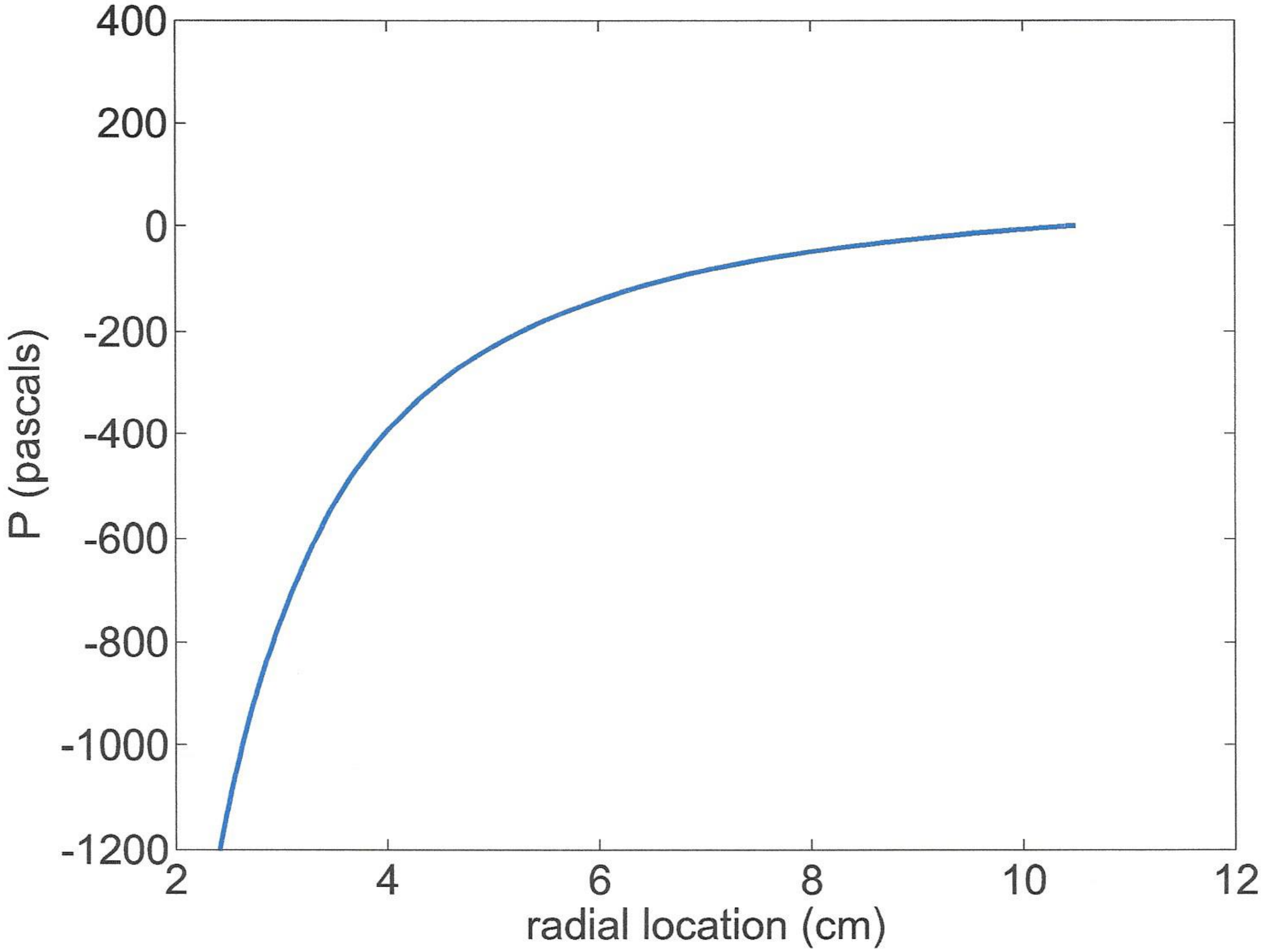
$$P + \frac{1}{2}\rho V_r^2 = P_\infty + \frac{1}{2}\rho V_R^2$$

$$P - P_\infty = \frac{1}{2}\rho\left(V_R^2 - V_r^2\right) = \frac{1}{2}\rho\left(\frac{Q^2}{4\pi^2 r^2 H^2} - \frac{Q^2}{4\pi^2 R^2 H^2}\right)$$

$$P - P_\infty = \frac{1}{2}\rho\frac{Q^2}{4\pi^2 H^2}\left(\frac{1}{R^2} - \frac{1}{r^2}\right)$$

See plot on next page. Not bad!

Bernoulli fails when plate is floating like an air hockey puck. Bernoulli always predicts low pressure. Need to include viscosity to model the flow.



400
200
0
-200
-400
-600
-800
-1000
-1200
P (pascals)
2
4
6
8
10
12
radial location (cm)