Bernoulli's equation

See the numbered schematics below which are associated with the following problems. In all problems here assume that the Reynolds number is high and the effect of viscosity is negligible. Apply Bernoulli's equation along a streamline to answer these questions;

$$P + \frac{1}{2}\rho \mathbf{v}^2 + \rho g z = \text{constant along a streamline.}$$

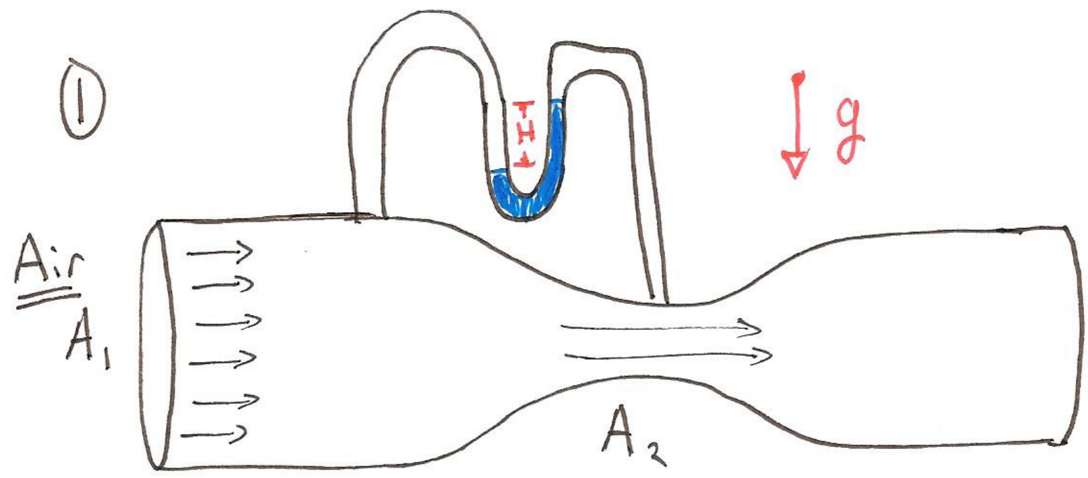
Recall that Bernoulli's equation assumes incompressible, steady, and inviscid flow. In some of the problems you will also need to apply conservation of mass; e.g. that the flow rate through the system is constant.

1. Consider air flow through a venturi meter at an unknown volumetric flow rate Q. The cross sectional area of the inlet and contraction are known. The height of water in the manometer is measured. Derive a relationship for the flow rate as a function of the measured height.

2. Air flows through a duct as shown. Determine the velocity of the air as a determined by the measured height of water in the manometer.

3. From the measurement of the height of water before and after a sluice gate, estimate the total flow rate through the open channel.

4. The inlet to a venturi is at atmospheric pressure. Air flows through the venturi at flow rate Q_a. A small tube is connected to a tank of liquid through a thin pipe. The liquid is drawn into the venturi by the low pressure at the throat. The liquid flow rate, Q_l, can be computed through Poiseueille's law ($\Delta P = Q128\mu L/(\pi D^4)$). Derive a formula for the flow rate of liquid in terms of the flow rate of air, the fluid properties of air and the liquid, the area contraction ratio of the venturi, and the length/diameter of the liquid pipe.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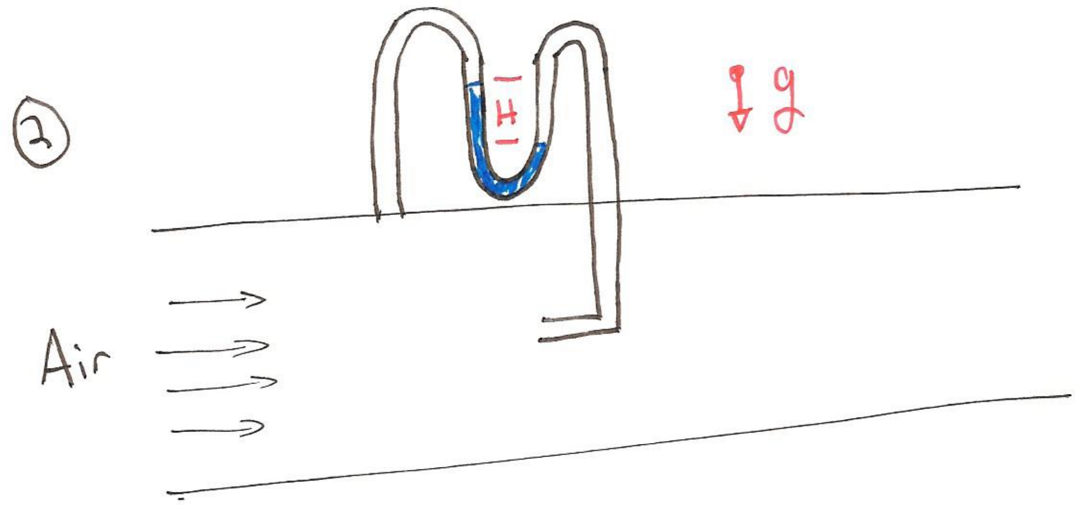
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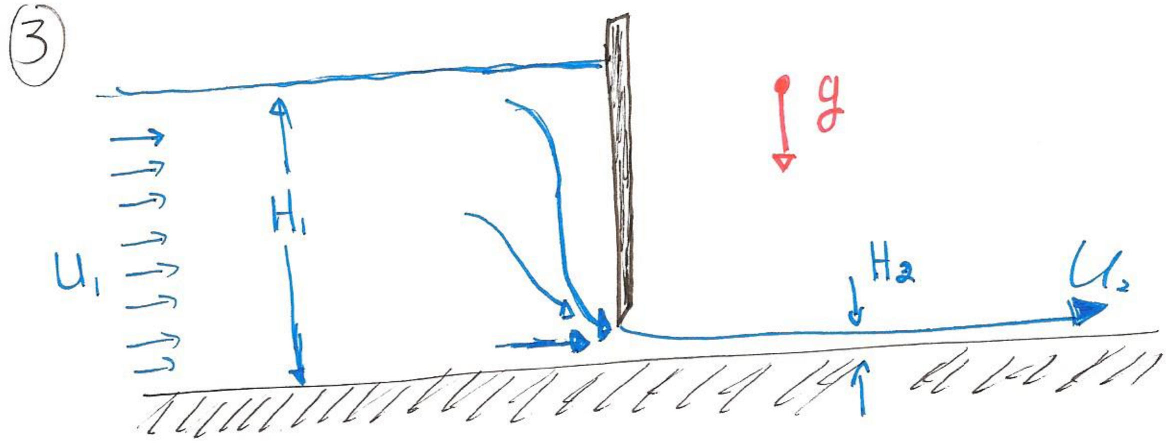
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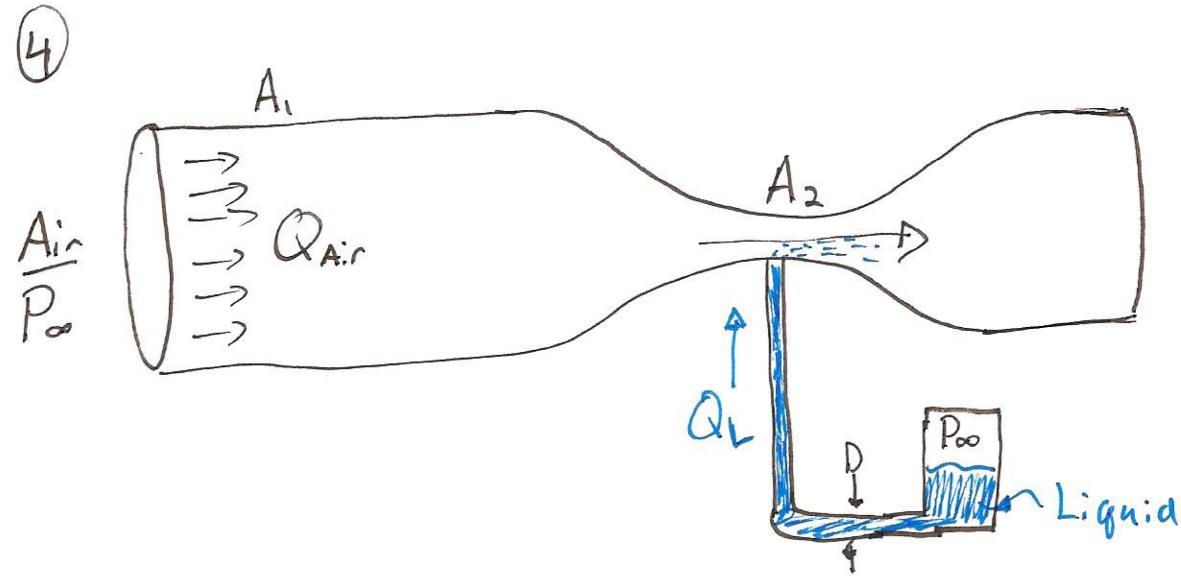
④
A_1
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