## Vector calculus

1. Consider the scalar field,

$$
f=\exp \left(-10\left(x^{2}+y^{2}\right)\right)
$$

- Sketch by hand the contours of the scalar field on the domain $-1<x<1,-1<y<1$.
- Compute the gradient and sketch the vector field $\nabla f$. You can use wolfram alpha.
- Compute the Laplacian $\nabla^{2}$. You can use wolfram alpha.

2. Consider the vector function given by,

$$
\mathbf{f}=x \mathbf{i}+y \mathbf{j}
$$

- Skecth by hand the vector field.
- Compute the divergence.
- Consider a circle of radius $R$ centered at the origin; compute $\mathbf{f} \cdot \mathbf{n}$.
- Evaluate $\int \mathbf{f} \cdot \mathbf{n} d S$ along the same circle.
- Evaluate $\int \nabla \cdot \mathbf{f} d V$ for the same circle.
- Consider a square of side length L centered at the origin, $\int \mathbf{f} \cdot \mathbf{n} d S$ along the boundary of the square.
- Convert the vector field to one described in cylindrical coordinates.
- Look up the divergence operator in cylindrical coordinates and recompute the divergence of the field.

3. Consider the vector function given by,

$$
\mathbf{f}=-x \mathbf{i}+y \mathbf{j}
$$

- Sketch the vector field.
- Compute the divergence.
- Consider a square of of side $L$ centered at the origin; compute $\mathbf{f} \cdot \mathbf{n}$.
- Evaluate $\int \mathbf{f} \cdot \mathbf{n} d S$ along the same square.

4. Demonstrate the

- $\nabla \times(\nabla \phi)=0$ where $\phi$ is a scalar field.
- $\nabla \cdot(\nabla \times \mathbf{v})=0$ where $\mathbf{v}$ is a vector field.

5. The operator $\mathbf{v} \cdot \nabla$ where $\mathbf{v}$ is the vector field of the fluid velocity will show up a lot in our equations in this course. Expand the operator out in component form where the components of velocity are $\mathbf{v}=[u, v, w]$ for the $x, y$, and $z$ directions.
