HW 3: Basic pipe flow

I. FLOW IN PIPES

Through dimensional analysis, we can figure out the general form of the pressure drop needed to force fluid through a tube. The pressure drop is usually written as

$$\Delta P = \frac{1}{2}\rho U^2 f \frac{L}{D}$$

where f is the friction factor and is a function of the Reynolds number. Note that this functional form is equivalent (though it may not appear at first glance) as that in Chapter 2. When the Reynolds number is less than about 2000, the flow is smooth and laminar and the friction factor can be found analytically (later is this course)

$$f = \frac{64}{Re}.$$

The Reynolds number is defined using the tube diameter as the appropriate length scale and uses the average velocity, U. The average velocity is related to the volumetric flow rate as $Q = \pi R^2 U$. When the Reynolds number is larger than 2000 the flow becomes turbulent and complex and f is found by use of the Moody chart which is based on experimentally measured data. For turbulent flows, f is a function of the Reynolds number and the roughness of the pipe.

- Show that the functional form given above for laminar flow (i.e. $f = 64/Re$) is equivalent to the often used Poiseuille's law

  $$\Delta P = QR$$

  where Q is the volumetric flow rate and R is the hydraulic resistance given by

  $$R = \frac{128\mu L}{\pi D^4}.$$

  Note that this law only works for Reynolds number less than ~ 2000. "Derive" the form of Poiseuille's law using dimensional analysis; recall dimensional analysis cannot tell you the factors of 128 and π but it can give you the dependence on length, diameter and viscosity.

- In class you will have taken data on the flow rate through several tubes of different lengths, diameters and applied pressures. You will have around 10 different data points. For each measurement, record the flow rate, the tube length, the tube diameter, and the height difference between the top of the tank driving the flow and the tube exit. For your homework, you can make a simple table to compare your experimental flow rates to those predicted by analysis based on the Moody Chart approach. For a few of the tubes compare the flow rate for a straight and coiled tubes, all other variables being equal. Comment on how bends and curves in the pipes influence the overall flow resistance.

- How much pressure is needed to deliver water through a 1/2 inch diameter pipe (typical in home plumbing) at 1 gallon per minute over a length of 50 ft of pipe. Convert your pressure to equivalent height - i.e. hydrostatic pressure is given as $\rho g h$. Now assume the pipe made 6 right angles. How much extra pressure drop do the bends induce? Not for the last part you will need to look up losses typical in 90 degree bends.

- Typical water pressure supplied to a home is around 50 psi. At this inlet pressure, what would be the flow rate through 50 feet of 1/2 inch diameter pipe, assuming no other losses?

- The friction factor has numerous empirical fit functions that one can find in the literature. A simple one was proposed by Moody,

  $$f = 0.0055\left(1 + \left(2\cdot 10^4 \frac{\epsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right).$$

  Plot this function along with the solution for laminar flow, $f = 64/Re$, as a function of Reynolds number and compare by eye to the Moody chart for a few pipe roughness values. Using this formula for the friction factor when $Re > 2000$ and the exact solution for laminar flow when $Re < 2000$, generate a plot of the pressure drop as a function of flow rate in a smooth pipe - assume again 50 ft of 1/2 inch copper pipe. Plot the result on both linear and log scales.

- Look up how to read pump curves. You can find some short explanations online. Qualitatively explain (in a sketch, for example) how you go about selecting a pump to deliver a desired flow rate through the simple piping system of the previous problem.

- In various applications of microfluidics we often need to force relatively high flow rates through narrow channels. Taking a channel to be 40 microns in diameter and 30 mm in length, what is the require pressure to deliver 10 ml/hour of water (a high flow rate for such a small channel). If the flow is delivered via a standard 10 ml syringe with a diameter of 14.5 mm, what force is applied to the syringe? Note that in applications, though small channels are usually made square the formula for a circular tube of the same size works well enough for estimation purposes.

- Human capillaries are about 6 microns in diameter. Average flow velocities in capillaries are about 1 mm/s. What is the pressure drop across a 1 mm long capillary? Compare this drop to typical blood pressure. For now, you can ignore the extra resistance due to the red blood cells and just assume that the plasma is close to water. The heart pumps about 6 liters per minute. Based on this total flow rate and the fact that capillary flow rates are about 1 mm/s, estimate how many parallel capillaries there are. Note that the main pulmonary artery has an internal diameter about 2.5 cm - compare the total cross sectional area of all the capillaries to that of the primary artery.