Control volumes

1. Water flows over a flat plate. At the leading edge of the plate there is uniform velocity U_∞. At the trailing edge we measure the velocity boundary later that has developed across the plate, $u(y)$. We measure the velocity as a function of y, the distance from the plate. The boundary layer has the conditions that $u(y=0)=0$ and $u(y>d)=U_\infty$. Using an appropriate control volume, measure the drag force on the plate as a function of the free stream velocity, the measured velocity profile, the boundary layer thickness, an any relevant fluid properties. Your answer will be in terms of an integral which you could evaluate numerically if you made the measurement.

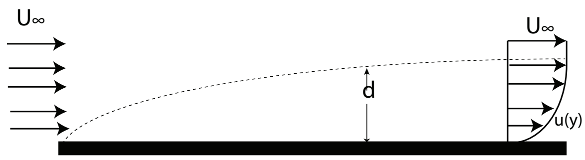

FIG. 1 Schematic for problem 1.

2. The hydraulic jump can be observed in the bottom of a sink. Notice that as the water spreads radially, that there is point where the height of the water suddenly increases. This jump can be observed in the spillways of dams and in rivers. The transition from height h_1 upstream can be related to the height h_2 downstream. Assume the upstream velocity, U_1, and the downstream velocity U_2 are constant across the layer depth. Determine the relationship between the upstream and downstream heights. Try to rearrange everything such that $h_2/h_1 = f(Fr)$ where the Froude number is, $Fr = U_1/\sqrt{gh_1}$.

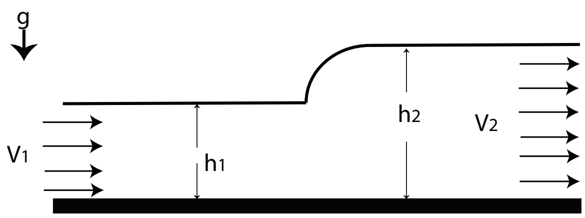

FIG. 2 Schematic for problem 2.

3. A jet pump is shown below. Find the velocity of the exit in terms of V_1, V_j, and A_j/A. Find the pressure drop over the pump. Is the exit at a higher or lower pressure than the inlet? Assume the flow is uniform as shown. Once you have a general solution, take $V_j = 2V_1$ and $2A_j = A$ - compute the pressure change.

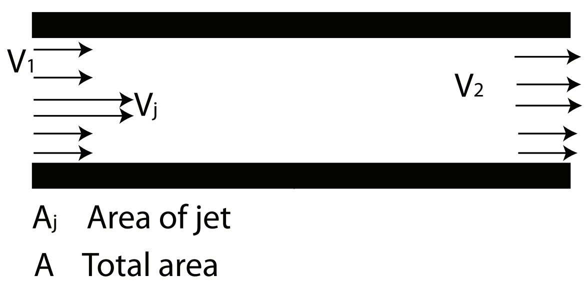

FIG. 3 Schematic for problem 3.

4. Two water hoses are aimed at each other and collide at a right angle (jets 1 and 2). The jets exit the hose, collide in the air, and form a single homogeneous jet as shown (jet 3). Ignoring gravity and assuming the flow velocities (V) are constant across the jet areas (A),

   - Set up the problem with enough equations to compute the velocity, area, and angle of the resulting jet 3 from the incoming jets, 1 and 2.
   - Solve for the angle of the resulting jet in terms of V_1, A_1, V_2, and A_2. Check the limiting cases of jet 1 dominates, jet 2 dominates, and jet 1 and 2 are equal.

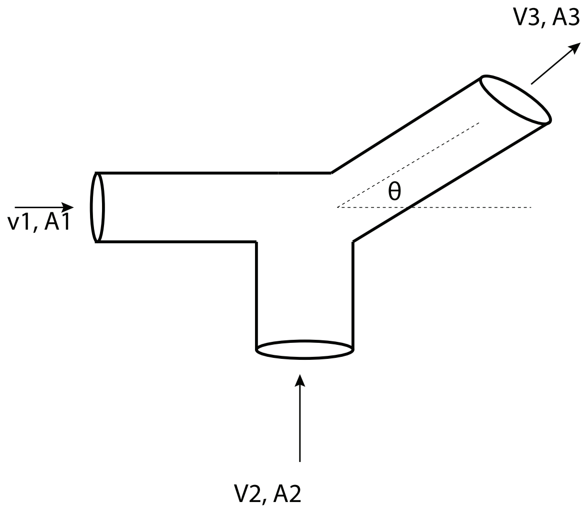

FIG. 4 Schematic for problem 4.

5. A gate holds back water of height H as shown in the figure below. When the gate is closed, the horizontal force (per unit depth into the page) needed to hold the gate in place is $F = \frac{1}{2}\rho g H^2$, since the pressure increases linearly with depth. When the gate is open, water flows out of a relatively small gap of height h. When the water is flowing, does the force needed to hold the gate increase, decrease, or stay the same? Prove it. Does your result change if you account for or neglect viscosity? Please note I am only asking about the trend, you don't have to actually calculate the force!

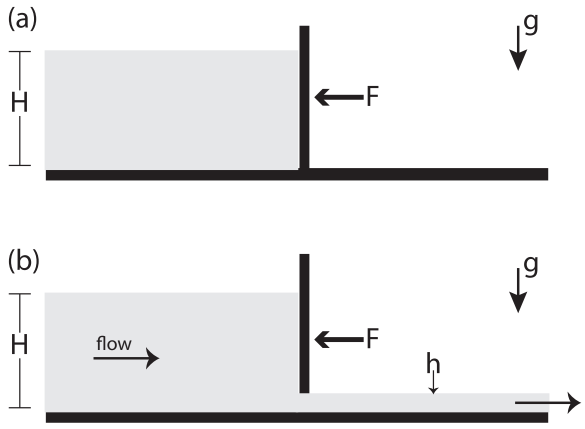

FIG. 5 Schematic for problem 5.