Inviscid flow continued, boundary layers

I. VORTEX DYNAMICS

The vorticity, ω, is related to the solid body rotation of infinitesimal fluid particles. The vorticity is given as the curl of the velocity field, $\nabla \times \mathbf{v}$. The 2D ideal vortex has a velocity field in radial coordinates given as

$$v_\theta = \frac{\Gamma}{2\pi r}$$

- Show that the circulation around this vortex is given by Γ.
- Show that, except at the origin of the vortex, the vorticity everywhere else is zero.
- Since the vorticity is zero everywhere except at the origin of the vortex, the velocity field can be shown to follow a potential. When $\nabla \times \mathbf{v} = 0$ we can define the velocity potential, ϕ. The potential is related to the velocity through $\nabla\phi = \mathbf{v}$. Prove that we can define a velocity potential by showing that $\nabla \times \nabla\phi = 0$. You can do this simple vector calculus proof by carrying out the operation component by component.
- Find the velocity potential which gives the ideal vortex velocity field.
- Since the velocity is defined by $\nabla\phi = \mathbf{v}$, show that the equation for fluid motion is $\nabla^2\phi = 0$. What's nice about this result is that the equation is linear, thus when flow has no vorticity we can add up the velocity potentials for different simple flows to get a more complex one. We are allowed to use superposition.

In 2D the vorticity equation is

$$\frac{D\omega}{Dt} = 0$$

The interesting thing about this equation is that it says that vorticity is locked in the fluid and just goes with the flow. Vorticity of individual fluids particles does not change. This means that if we have a single ideal point vortex, it sits there spinning and the vortex cannot move itself. If we have two vortices, since the velocity field away from the origin is irrotational, the velocity field of each vortex adds up everywhere in the fluid. The velocity field of one vortex moves the other at its center and vice-versa. To simulate point vortices, we can write a very simple program using these facts.

We go to each point vortex in the simulation. We compute the velocity of that vortex's center by adding up the velocity field of all other vortices. We can then move the vortex in simulation a small distance using Euler integration, i.e. new position is current position plus velocity times time. We then update all vortex positions and move on. We will review this method in class.

Write a simple program to simulation the motion of vortices. Confirm your simulation with two vortices. The two vortices should move each other in a straight line when given the same strength, but opposite spin directions. The two vortices should orbit each other in a circle if given the same strength and the same direction. If you are feeling good about this, try to extend to four vortices and see if you can simulate the approach of two vortices with a wall as discussed in the book.

II. BOUNDARY LAYERS

1. A passenger plane is flying at cruise altitude of 30,000 ft. How far from the leading edge of the wing does the boundary layer become turbulent, if at all? Considering the flow along the fuselage, what is the boundary layer thickness on the fuselage near the back of the plane? These are estimates so be clear in what you are assuming for size, speed, etc.
2. A car is traveling at 35mph (15 m/s). Consider the flow on the hood of the car. How far from front does the boundary layer become turbulent, if at all ? *Estimate* the boundary layer thickness over the length of the hood.
3. For water flowing at a meter per second over a long flat plate, how far down does the flow transition to turbulence. Make a dimensional plot of boundary layer thickness versus distance to get a feel for the order of magnitude of numbers involved. Your plot should include the overlap of the laminar to turbulent transition.
4. A one square meter plate is held horizontal to the water flow at 1 m/s. Compute the total drag force acting on the flat plate. Consider whether the flow is laminar, turbulent, or mixed when computing your answer.