# Vector calculus

1. Consider the scalar field,
$$f = \exp(-10(x^2 + y^2))$$
   - Sketch by hand the contours of the scalar field on the domain $-1 < x < 1$, $-1 < y < 1$.
   - Compute the gradient and sketch the vector field $\nabla f$. You can use wolfram alpha.
   - Compute the Laplacian $\nabla^2$. You can use wolfram alpha.
2. Consider the vector function given by,
$$\mathbf{f} = x\mathbf{i} + y\mathbf{j}$$
   - Skecth by hand the vector field.
   - Compute the divergence.
   - Consider a circle of radius R centered at the origin; compute $\mathbf{f} \cdot \mathbf{n}$.
   - Evaluate $\int \mathbf{f} \cdot \mathbf{n} dS$ along the same circle.
   - Evaluate $\int \nabla \cdot \mathbf{f} dV$ for the same circle.
   - Consider a square of side length L centered at the origin, $\int \mathbf{f} \cdot \mathbf{n} dS$ along the boundary of the square.
   - Convert the vector field to one described in cylindrical coordinates.
   - Look up the divergence operator in cylindrical coordinates and recompute the divergence of the field.
3. Consider the vector function given by,
$$\mathbf{f} = -x\mathbf{i} + y\mathbf{j}$$
   - Sketch the vector field.
   - Compute the divergence.
   - Consider a square of of side L centered at the origin; compute $\mathbf{f} \cdot \mathbf{n}$.
   - Evaluate $\int \mathbf{f} \cdot \mathbf{n} dS$ along the same square.
4. Demonstrate the
   - $\nabla \times (\nabla \phi) = 0$ where $\phi$ is a scalar field.
   - $\nabla \cdot (\nabla \times \mathbf{v}) = 0$ where $\mathbf{v}$ is a vector field.
5. The operator $\mathbf{v} \cdot \nabla$ where $\mathbf{v}$ is the vector field of the fluid velocity will show up a lot in our equations in this course. Expand the operator out in component form where the components of velocity are $\mathbf{v} = [u, v, w]$ for the $x$, $y$, and $z$ directions.