Equations for heat conduction

The conservation of energy for heat conduction in a solid is written in integral form as,

$$\frac{d}{dt}\int \rho C T dV = -\int \mathbf{q}\cdot\mathbf{n} dS,$$

and in differential form as,

$$\rho C\frac{\partial T}{\partial t} = -\nabla\cdot\mathbf{q},$$

where $\mathbf{q}$ is the heat flux vector, k is the material's thermal conductivity, C is the specific heat, and ρ is the mass density. We will always assume that the density and specific heat are constants. The heat flux vector is related to the temperature field through Fourier's law

$$\mathbf{q} = -k\nabla T.$$

When the conductivity is constant the heat equation becomes,

$$\frac{\partial T}{\partial t} = \alpha\nabla^2 T.$$

where the thermal diffusivity α is defined as $\alpha = k/\rho C$. Written in component for cartesian coordinates, rather than vector form, the Laplacian is expanded as

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right).$$

To solve conduction problems we require boundary conditions at the surface of the volume of interest. Common boundary conditions include a fixed temperature, insulation $\mathbf{n}\cdot\mathbf{q} = 0$, or convection $\mathbf{n}\cdot\mathbf{q} = h(T - T_\infty)$ where h is the empirically determined convection coefficient.

In heat transfer problems with convection, the most prevalent parameter is the Biot number defined as $\mathrm{Bi} = hL/k$.