Quiz 2, Your name:

Please write your name. Please write your final answers directly on paper in the space next to the problem. Attach any additional work at the end. You may use up to three hours. You may use your book and notes. You may use a calculator, or for the estimates feel free to round up/down to easy to calculate numbers by hand (just denote what you assumed).

I. CONCEPT QUESTIONS

1. Conceptually, when we write the material derivative of temperature, DT/Dt, what does that represent?

2. Identify the physical meaning of each of the terms in the momentum equation. Not a long explanation - just a short note what each term is.
   $\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla P + \rho\mathbf{g} + \mu\nabla^2\mathbf{v}$.

3. Consider a 2D flow in the x-y plane where u is the x component of velocity and v the y component. Consider a flow that has a velocity gradient such that $\partial u/\partial x > 0$. Consider a fluid particle that was a perfect square at time=0. Sketch the fluid particle a short time later.

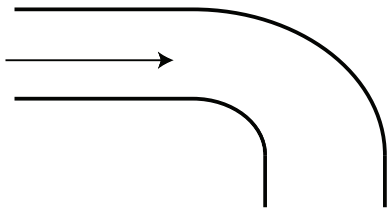
4. How is the Reynolds number defined and what is the physical interpretation of it?

5. What assumptions must hold for Bernoulli's equation to be valid?

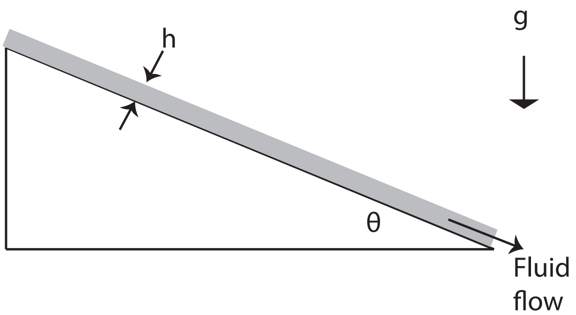
6. What boundary conditions are applied to the Navier-Stokes equations at a stationary solid surface?

7. What boundary conditions are applied to Euler equations at a stationary solid surface? Recall that Euler's equations ignore viscosity.

8. Consider inviscid flow around a 90 degree bend. Is the pressure higher, lower or the same at the inside of the bend relative to the outside? (denote on the figure).



9. Consider a thin layer of liquid flowing down a ramp inclined to gravity. The liquid is being continuously poured at the top of the ramp at a constant rate. The surface of the liquid film is open and exposed to air and the film has a constant height. SKETCH the velocity profile in the liquid. Be VERY clear what the velocity profile is doing (qualitatively) at the wall and free surface. Please draw a zoomed in version of the velocity field in the thin film, don't superimpose it on the figure below.



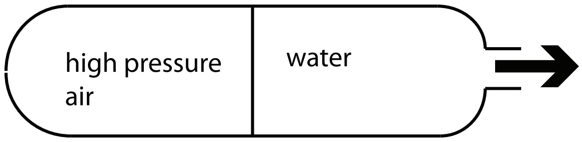
10. Hot water flows into a pipe. The wall of the pipe is held at a fixed constant temperature. Sketch, qualitatively, what the mean temperature of the water is as a function of distance down the pipe.

II. SOME SIMPLE ESTIMATES

1. You are interested in helping your 10 year old cousin win a soda-bottle water-rocket contest at her school. Being a good engineer, you help her build a test stand to measure the thrust of different designs. As you are testing the rocket, you become confident in your fluid mechanics skills and think that you can predict the thrust. The air is initially pressurized 1 atm above the ambient pressure. The bottle opening has a diameter of 28 mm. The two liter bottle is initially half filled with air and half filled with water.

   - What is the estimated velocity of the water jet at this pressure?
   - What is the estimated initial thrust force?

   Note: This is just an estimate for the instant that the rocket is fired. You do not need to worry about the change in pressure with time as the bottle discharges.

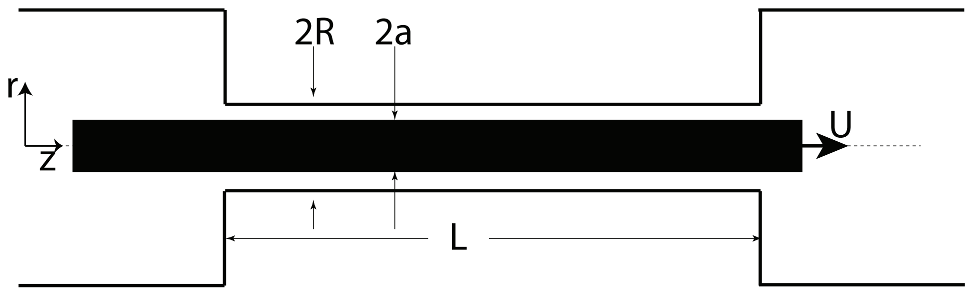


2. Estimate the pressure needed at a pumper truck in order to spray water onto the roof of a five story building whose roof is approximately 50 ft (15 meters) above street level.

3. Water ($\rho = 1000$ kg/m^3, $C = 4186$ J/kgK, $k = 0.6$ W/mK) at 25 C flows over a 10 cm diameter sphere held at a constant temperature 50 C. Using an empirical correlation for our flow condition we find the Nusselt number is about 15. What is the heat transfer coefficient, h? What is the total heat transfer rate from the sphere to the fluid in watts?

III. ONE SLIGHTLY LONGER PROBLEM

1. Consider a solid cylindrical rod which is concentrically located in a fluid filled tube. The rod is pulled at a constant speed U. The rod has radius a and the tube radius R. **The reservoirs at the ends of the tube are held at the same pressure**. The tube is long and the gap between the cylinder and tube is relatively thin, such that we can consider the flow to be the same at any axial location. Assume everything is axisymmetric and at steady state. Gravity is normal to flow and has no effect. As always, we are only concerned with incompressible, Newtonian flow. Using the assumptions of axisymmetric, steady (no time dependence), and no gravity, the equations of conservation of mass and momentum in cylindrical coordinates is known as,



$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0,$$

$$\rho\left(v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{\partial^2 v_r}{\partial z^2},\right)$$

$$\rho\left(v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}.\right)$$

where the notation, v_r refers to the radial velocity and v_z refers to the axial velocity. The radial coordinate is r and the axial one is z. The radial coordinate r is centered on the center of the rod.

- Using the further assumptions that the flow is constant down the length of the pipe, cross out all the terms that are zero. Reduce the above set of equations to a single, much much simpler, equation.

- State the boundary conditions that should be applied at $r = a$ and $r = R$.

- We would like to know the total force needed to maintain constant speed. We could do this exactly by integrating your equation above and evaluating the viscous stress at the rod's surface. But we're lazy. Let's just assume that the gap between the rod and the tube is thin such that the flow in the gap will be approximately the same as that found in Couette flow. What is the APPROXIMATE force needed to maintain constant speed. There is no partial credit for this answer if your units are incorrect!