Quiz 1

I. DIMENSIONAL ANALYSIS

1. Imagine a sphere of diameter, D, traveling at constant velocity U, in a fluid of viscosity μ. (μ is the dynamic viscosity and has units of Pascal-seconds).

   For small objects at very low speeds (i.e. a small organism, a speck of dust) the fluid's inertia is unimportant and thus its mass density is observed not to influence the drag force. Derive the form for the drag force on a sphere in the limit where viscosity is the dominant force.

2. Consider the previous problem. The sphere is dropped in the viscous fluid, comes to terminal velocity very quickly, then descends down a container of a certain depth. If you double the diameter of the sphere (keeping the material and all other properties constant), how does the time to reach the bottom change?

II. VECTOR CALCULUS

1. A vector field is given as $\mathbf{f} = x\mathbf{i} + y\mathbf{j}$. Compute the divergence of $\mathbf{f}$.
2. A scalar field is given as $f = \sin(x)y$. Compute the gradient of f.
3. State the divergence theorem mathematically.

III. HEAT TRANSFER

1. Explain in words what the Biot number represents.
2. Two spheres are cooling in air. The Biot number is computed and found to be very small for both. Sphere A has a thermal conductivity 10 times sphere B. All other properties are the same. Does A cool faster, B cool faster, or do they cool at about the same rate?
3. Two spheres are cooling in air. The Biot number is computed and found to be very large for both. Sphere A has a thermal conductivity 10 times sphere B. All other properties are the same. Does A cool faster, B cool faster, or do they cool at about the same rate?
4. A hot sphere 1 cm in diameter is left to cool in air. The convection coefficient is experimentally measured to be 10 W/(m^2K). The properties of the material are density $\rho = 1000$ kg/m^3, thermal conductivity of $k = 100$ W/(mK), and specific heat of $C = 1000$ J/kgK.
   - Estimate the Biot number for this problem.
   - Sketch the temperature field as a function of radius at different instances in time.
5. A hot sphere 10 cm in diameter is left to cool in air. The convection coefficient is experimentally measured to be 10 W/(m^2K). The properties of the material are $\rho = 1000$ kg/m^3, thermal conductivity of $k = 0.1$ W/(mK), and specific heat of $C = 1000$ J/kgK.
   - Estimate the Biot number for this problem.
   - Estimate how long the sphere will take to cool.
6. Consider an object being cooled at its surface by convection with coefficient h. Assume we are in a regime where the objects temperature is approximately uniform throughout. From conservation of energy in integral form (see equation sheet), derive the lumped model which provides the time rate of change of the objects temperature.
7. In an object, I measure the temperature field to be approximately $T(x, y, z) = xy$. What is the heat flux?
8. Consider a square domain as shown in Figure 1. Two adjacent sides are held at a constant temperature, one is hot the other one cold. The other two sides are insulated. At steady state, sketch lines of constant temperature and heat flux vectors. Be clear with behavior at the walls.

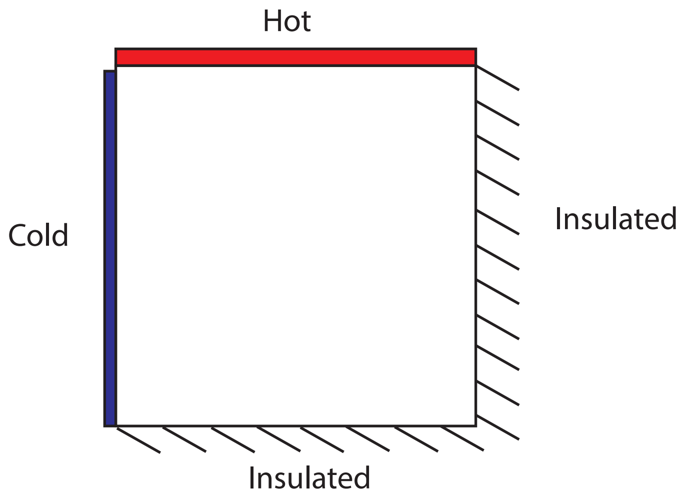

FIG. 1 Figure for problem 8 on heat transfer

IV. NO EXTRA CREDIT PROBLEMS

1. The pace of the course is too fast, too slow, or about right.
2. The workload is too much, too easy, or about right.