Fluid motion

I. READING

This week you should watch

- Videos for chapters 5, 6, & 7

II. READING

This week you should read

- Chapter 5, 6 & 7

III. MASS TRANSFER

1. Write a short MATLAB program to simulate the 1D random walk of a set of particles. Locate 10,000 particles at the origin $x = 0$. At each time step move the particles a distance proportional to a normally distributed random number (use the `randn` command). Use the MATLAB vector notation to keep this simple. For example to update the particle position at each time step you can simply do `x = x + dt*randn(size(x));` where dt is a scaling factor. Use the `hist` command to plot the distribution of particles. Compare the result to the analytical solution based on the diffusion equation which is

$$c(x,t) = \frac{C}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

   where $C = \int_{-\infty}^{\infty} c(x,t)dx$ is the total amount of stuff. While a one to one comparison is possible, you can just confirm the result qualitatively.

2. These are estimates to get a sense for some orders of magnitude.
   - The diffusivity of dissolved oxygen in blood is about $2 \times 10^{-9}\mathrm{m}^2/\mathrm{s}$. How long does it take oxygen to diffuse over a distance of 5 microns (the scale of capillaries) ? 1 micron (the barrier between oxygen and blood in the lungs)?
   - Typical diffusion constants for sugar in water are $5 \times 10^{-10}\mathrm{m}^2/\mathrm{s}$. How long would it take for diffusion to mix sugar in a cup of water.
   - Typical diffusion constants for acetone in air are around $1.2 \times 10^{-5}\mathrm{m}^2/\mathrm{s}$. How long would it take you to smell a bottle after opening if there were only molecular diffusion.

IV. READING QUESTIONS

1. Write out, in component form, Dc/Dt, where c would be the scalar concentration field.
2. What is Fick's Law?
3. Explain, in words what the material derivative represents.
4. Why do we use a tensor to represent the state of stress of a material?
5. What is the physical meaning of the divergence of the stress tensor?
6. What is the differential form of conservation of mass for an incompressible flow?
7. When is the assumption of incompressible flow a good one?
8. What role does gravity play in a constant density flow?

9. What is the relationship between the stress tensor at a point and the stress vector acting on a surface passing through that point?
10. What's the difference between Euler's equations and the Navier-Stokes equations?
11. Physically, what is the reason that the stress tensor must be symmetric?
12. Physically, conceptually, and briefly, describe in words what is the constitutive law for a Newtonian fluid.
13. What is the boundary condition on the fluid velocity at a solid wall (state in words or mathematically)?
14. What is vorticity and what does it represent?
15. What are the units of the stress tensor?

V. REYNOLDS NUMBER ESTIMATES

Estimate (order of magnitude) the Reynolds number for

- A car traveling at 55 mph.
- A passenger plane
- A hurricane (with 80 mph winds over 100 miles)
- A bacteria in water (2 μm in size moving at 100 μm/s)
- Make up 2 other things that involve fluid flow and estimate the approximate Reynolds number.