Week 3: Heat conduction

All problems due Tuesday, Sept 29, 2014.

I. READING

1. Read chapters 3 and 4.
2. Watch the video lectures on the heat equation.

II. PROBLEMS

1. State what 1 Joule AND 1 kilo-Watt hour (kWh) are in approximate terms of something you can remember. Come up with an example using potential energy, kinetic energy, thermal energy, and food. This exercise is just to try and provide a simple estimate that is easy for you to remember for what a unit of energy is. This can be useful for determining whether the energy changes we discuss are a "lot" or a "little".

2. A wall of thickness L and conductivity k separates the indoors from the outdoors. Imagine a case where the initially, the indoor air, the wall, and the outdoor air are all at the same temperature, T_{out}. The heat to the building is turned on such that the air inside is at T_{in}. Heat is transferred at the wall on the inside and outside by convection, with coefficient h_{in} and h_{out}. Write the equation, boundary conditions, and initial condition for the problem of what is the wall temperature distribution as a function of time. Make the formulation dimensionless and state the dimensionless problem mathematically. You don't need to solve the problem, just set it up mathematically.

3. Consider the previous problem. At **steady state**, solve for the temperature field inside the wall. Express in both dimensionless and dimensional terms. The result should be in terms of known quantities, such as the inside and outside air temperatures, the inside and outside convection coefficients, and the length of the wall. Sketch the temperature field for a few cases. Consider the cases where the inside and outside convection coefficients are equal and large, where they are equal and small, where the inside is large and the outside is small, and where the inside is small and the outside is large. Think about these results and make sure that you could explain them qualitatively.

4. For the previous problem at steady state, solve for the overall thermal resistance which accounts for indoor convection, conduction through the wall, and outdoor convection. The overall thermal resistance is that which follows, $T_{in} - T_{out} = qR$, where q is the heat flow (W/m^2).

5. Look up the conductivity (or R value) of typical wall insulation materials. Using convection coefficients (indoor and outdoor) of 10 W/m^{2}K and typical wall thickness, estimate typical values of heat loss (per unit area) on a cold winter day (25 C temperature difference). Look up typical numbers for the conductivity of windows and recompute. Note if you look up typical numbers online you will need to be very careful of units.

6. Consider a wall of thickness L. From $0 < x < L/2$ the conductivity of the material is k and from $L/2 < x < L$ the conductivity is $10k$. Consider that initially the temperature is T_c. At $t > 0$, $T(x = 0, t) = T_h$ and $T(x = L, t) = T_c$. Sketch the temperature profile qualitatively as a function of time. Pay attention to the final steady state temperature field. Assuming the two regions have the same density and specific heat, ρ and C, estimate how long it will take for the temperature field to come to steady state?

7. Consider steady state heat conduction through a wall of thickness L. The left and right boundaries have a fixed temperature; $T_h(x = 0)$ and $T_c(x = L)$. Consider a material whose thermal conductivity is a function which increases linearly with temperature; i.e. $k = k_o + A(T - T_0)$, where T_0 is a reference temperature, k_0 is the conductivity at the reference temperature, and A is an experimentally determined positive constant value of $A = dk/dT$ for the material. Qualitatively sketch the steady state temperature field compared to the case where the material has constant conductivity. Solve for the steady state temperature field.

8. Consider heat flow into a semi-infinite domain. The domain spans $0 < x < \infty$. Initially the temperature everywhere is T_0. At $t = 0$ the temperature at the boundary is suddenly increased to $T(x = 0, t) = T_h$. The solution to the dimensional problem is

$$\frac{T(x,t) - T_0}{T_h - T_0} = \mathrm{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right).$$

The term erf is known as the error function. MATLAB knows it. Wikipedia knows it. You can read about it. Plot the solution in dimensionless (just the equation above) and dimensional terms for the problem of soil in the earth responding to a sudden change in temperature at the surface. Look up reasonable properties for the thermal diffusivity of soil and show the evolution for 6 months. While the more realistic problem for the soil would have periodic forcing, the step response is just easier for us to write down the solution.

9. Consider the following dimensionless heat conduction problem,

$$\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial x^2}$$

with initial condition $\Theta(x, t = 0) = 1$ and boundary conditions $\Theta(x = 0, t) = 0$ and $\Theta(x = 1, t) = 0$. The analytical solution to this problem is

$$\Theta(t, x) = \sum_{n=1,3,5,..}^{\infty} \frac{4}{n\pi} \mathrm{e}^{-n^2\pi^2 t} \sin(n\pi x)$$

- Show that this function satisfies the equation.
- Show that this function satisfies the boundary conditions.
- Show that this function satisfied the initial condition (which is easiest to demonstrate numerically).
- Calculate the heat flux at $x = 0$ and write the result as a sum.