Heat transfer in fluids

I. VISCOUS HEATING

1. Consider pressure driven, plane parallel flow between two plates at steady state (Poiseuille flow). In this case, we will consider viscous heating and consider that the walls are held at constant temperature. When we take our domain to be $-H < y < H$ and $y = 0$ to be the centerline, the flow field (from conservation of mass and momentum) is

$$u(y) = \frac{\Delta P}{2\mu L}(H^2 - y^2),$$

and the total flow rate is

$$Q = \frac{2\Delta P H^3}{3L\mu}.$$

See section 8.1 of the book if you forgot this result. Now consider conservation of energy for a liquid. Under the assumption of steady state in time and equilibrium in the x direction, we have

$$k\frac{d^2T}{dy^2} = \mu\left(\frac{d\ u(y)}{dy}\right)^2 = \mu\left(-\frac{\Delta P y}{\mu L}\right)^2 = \frac{\Delta P^2}{\mu L^2}y^2.$$

- Starting with the last equation for conservation of energy in vector form at the end of section 13.2, justify/derive the simpler energy equation stated above.
- Integrate the temperature equation across the domain from $-H < y < H$ with constant temperature applied at the wall to solve for the temperature of the fluid across the channel, $T(y)$.
- Show that the total rate of heat leaving the fluid (in Watts) is equal to the hydraulic power applied to the fluid, $Q\Delta P$. Recall that the heat flux in the fluid going to the wall is given by $q = k\frac{\partial T}{\partial y}$, thus once the temperature field is found in the previous part it is easy to compute the heat flow.

II. EXTERNAL FLOW

1. Consider flow over a flat plate. Compute an average h for a number of different cases just to get a feel for the order of magnitude. Consider a plate the size of a piece of paper. Compute h for 0.1, 1, and 10 m/s. Compute for both air and water. It would be easiest to write a little MATLAB script or spreadsheet where you can put the formula in once and recompute the result. Provide your results for h as a 3x3 table for each fluid.

2. Use Comsol to confirm the empirical formula for h at a few select conditions and to visualize the temperature field, for the previous problem. Note that you will need to keep the Reynolds number from being too large in order to get an accurate solution from Comsol. It is probably easiest to compute the flow between two parallel plates and make the spacing large enough (but not too large that the computation takes a long time) that the thermal and viscous boundary layers don't interact much.

3. Consider a cylinder in cross flow. Compute h for a number of different cases just to get a feel for the order of magnitude. Change the diameter from 1, 10, and 100 mm. Change the flow speed from 0.1, 1, and 10 m/s. Consider both air and water. It would be easiest to write a little MATLAB script or spreadsheet where you can put the formula in once and recompute the result. Provide your results for h as a 3x3 table for each fluid.

4. Use Comsol to confirm the empirical formula for h at a few select conditions for the previous problem. Note that you will need to keep the Reynolds number from being too large (keep Re less than about 100 based on cylinder diameter) in order to get an accurate solution from Comsol.

5. Consider a heated cylinder in air cooled by natural convection at room temperature. Consider a cylinder of 1, 10, and 100 mm in diameter. Consider the cylinder is held hot at 10, 50, and 100 C above room temperature. Compute an average h for each case. It would be easiest to write a little MATLAB script or spreadsheet where you can put the formula in once and recompute the result. Provide your results for h as a 3x3 table.

6. Estimate h for a pizza fresh from the oven cooling on the counter.

III. INTERNAL FLOW

1. Consider a pipe 1.5 mm in diameter with water flowing at 1 ml/s. Water enters the pipe at 100 C. The pipe wall is held at 25 C. How long must the pipe be for the temperature of the water to exit at 30C? You can assume the pipe is long enough that the flow inside will be fully developed and thus the internal value of h is a constant.

2. Repeat the calculation but increase the pipe size to 6 mm and the flow rate to 100 ml/s. Again, you can assume the pipe is long enough that the flow inside will be fully developed and thus the internal value of h is a constant.

3. Imagine in the previous problem that the pipe is simply exposed to air and not held at a fixed wall temperature. External to the pipe, air flows around the pipe by natural convection and the air carries the heat away. Thus we have convection from the water to the pipe wall, conduction through the pipe, and convection external to the environment which is at a fixed temperature. Describe how you would solve for the exit temperature of the fluid. Assume that the pipe is thin walled copper and does not have an appreciable conduction resistance. You do not actually need to solve the problem, just formulate a simple, approximate method for doing so. Are there limits when either internal or external convection resistance dominates that the general problem simplifies?